JSJC : Probability team presentation

Kacem Lefki

4 novembre 2022

Stochastic processes

It is a family of random variables $(X_t)_{t \in T}$ (where $T = \mathbb{R}_+$ or $[0, a]$ almost everytime) valued in a set E (usually $E = \mathbb{R}$).

Mainly studied stochastic processes :

- Jump processes
- Continous processes

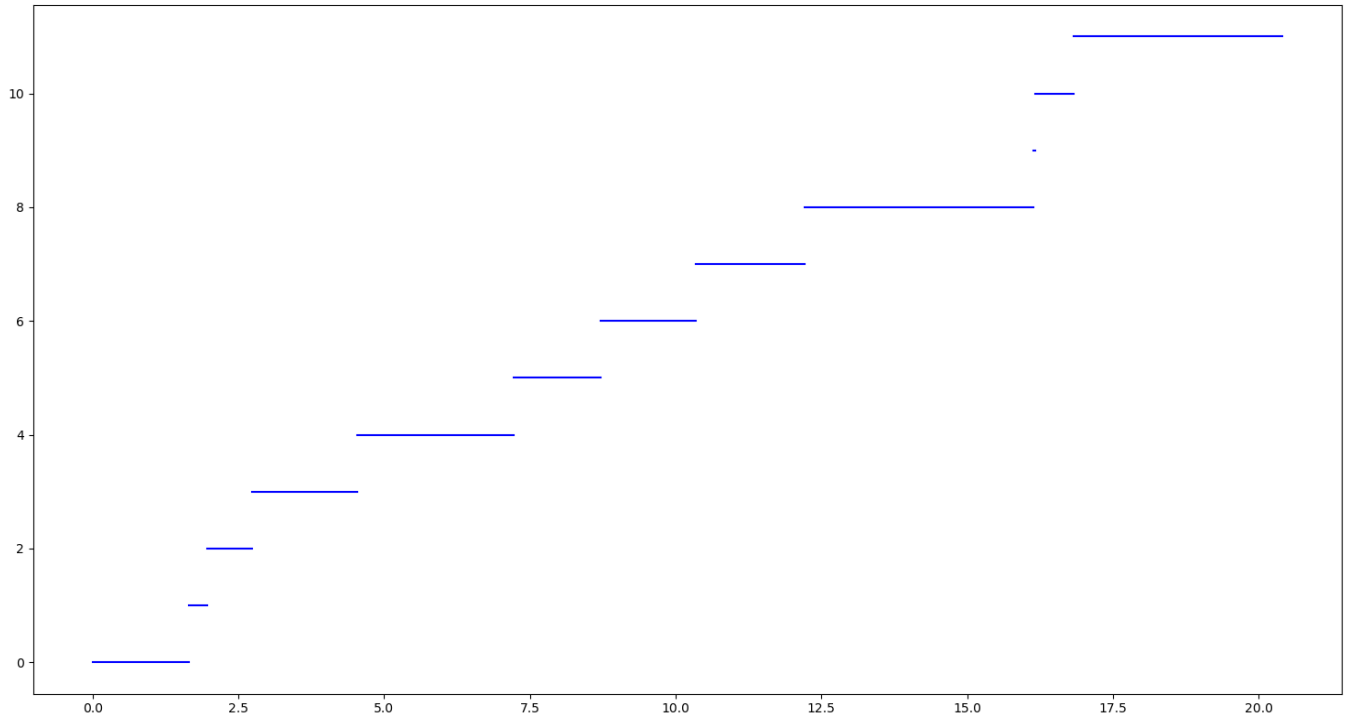
Poisson process

A process $(N_t)_{t\in\mathbb{R}_+}$ is a Poisson process of intensity $\lambda > 0$ if it fulfills the following conditions :

- $N(0) = 0$.
- Independent increments : $\forall 0 \leqslant t_1 < \cdots < t_n$, the random variables $N_i - N_{i-1}$ for $i = 1, \ldots, n$ are independant.
- Distribution of increments : $\forall 0 \leqslant t < s$, the random variable $N_s - N_t$ has a distribution $\mathcal{P}(\lambda(s-t))$.

In this case, if we denote $t_1, t_2, \ldots$ the jump instants, we have $t_{i+1} - t_i \sim \mathcal{E}(\lambda)$.

Poisson process

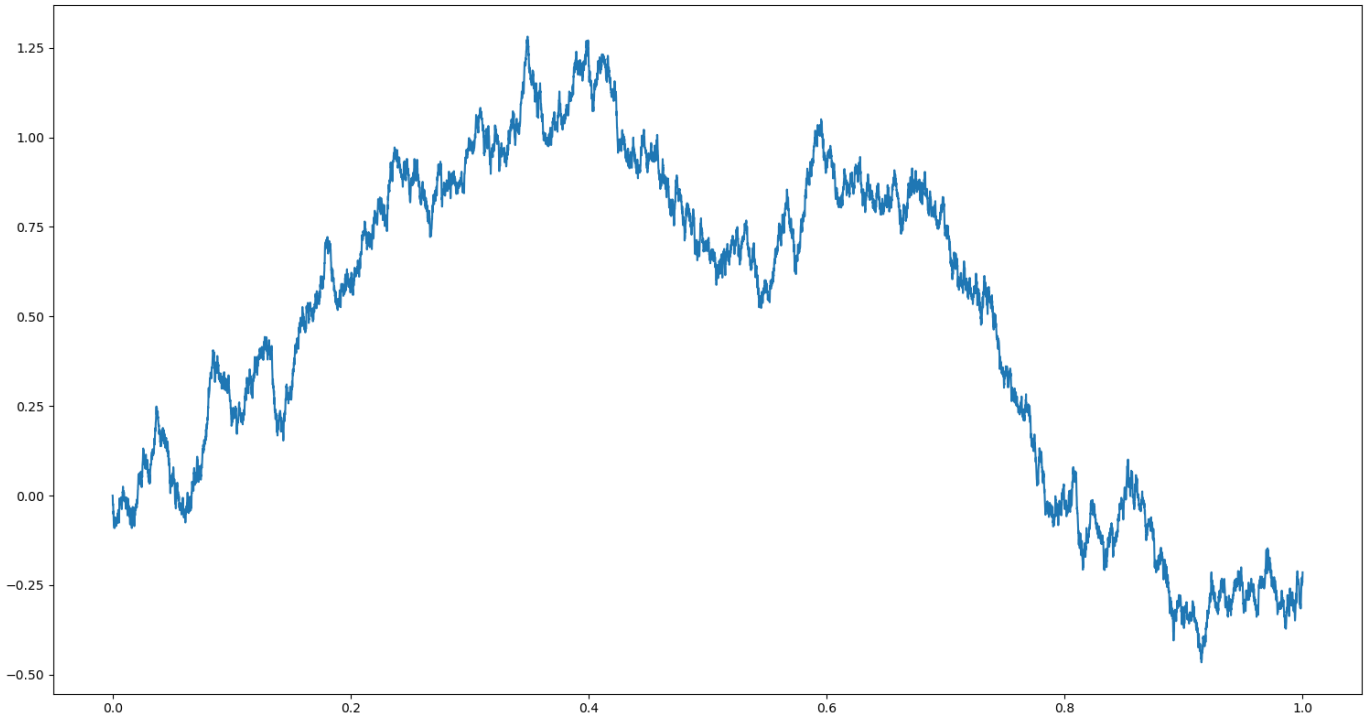


Brownian motion

$(X_t)_{t\in\mathbb{R}_+}$ is a brownian motion if it fulfills the following conditions :

- $B(0) = 0$.
- B is a continous function of the time t.
- Independant increments : For any $0 < t < s$, the random variable $B_s - B_t$ is independant of $(B_u)_{0\leqslant u\leqslant s}$.
- Distribution of increments : $\forall 0 \leqslant t < s$, $B_s - B_t \sim \mathcal{N}(0, s - t)$.

Brownian motion



Mathematical finance

Goal : Model and study financial markets, manage financial risks.
Exemple :

- When is it a good idea to buy/sell a stock ?
- Is it a good idea, for an insurance company, to insure a bunch of 30 young researchers going to Provins to "work" ? (Spoiler : no)

To be good at financial maths, one need to perfectly master stochastic differential equations and Brownian motion !

Permanent

Aurélien Alfonsi : Discretization of SDE, financial risk. Organizing the Groupe de Travail Méthodes Stochastiques et Finance (Edoardo will make a talk on Monday !). Member of the Mathrisk Team, common with INRIA, CERMICS and LAMA (UGE).

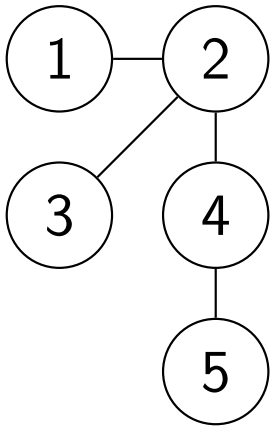
Benjamin Jourdain : Studies probabilistic numerical methods in finance (discretization of SDEs, variance reduction, ... and risk modeling in finance. Martingale optimal transport (see Coco's talk). Member of the Mathrisk Team.

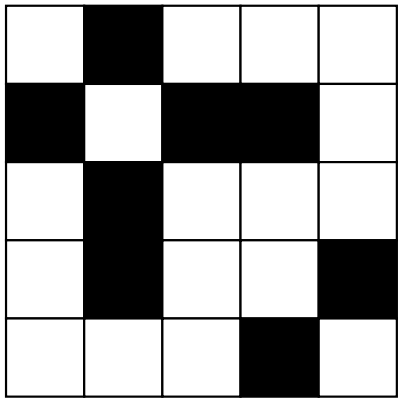
Julien Guyon : New professor at CERMICS, mainly studies nonlinear option pricing and stochastic volatility (a star according to Hervé).

Graphs

What is a graph ? Ask Hélène

Graphs



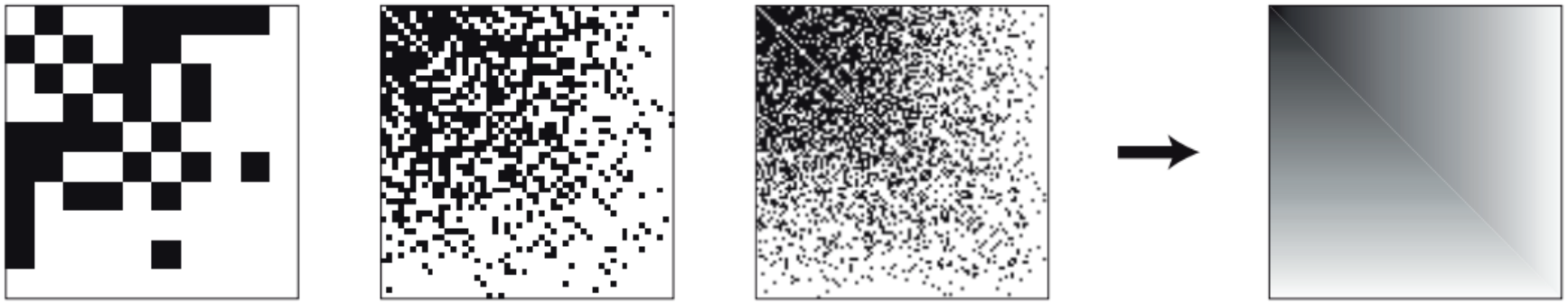


To get some probabilities : Edges are randomly chosen -> Random graph

Graphon

What would happen with the number of vertices is large ?

Figure – Convergence towards a graphon (By Daniel Glasscock in "What is ... a graphon ?")



Permanent

Julien Reygner : Stochastic processes, especially a long-time behaviour, metastability problems. Systems of particles in interaction.

Jean-François Delmas : Stochastic processes in biology (ex : epidemics modelling), and large random trees/graphs.