Optimization under Uncertainty

[Subtitle]

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Outline

Game

Option 1

Gain or lose between 10 and -10 euros.

Option 2

Gain or lose between 20 and -20 euros.

Game

Option 1

Gain or lose between 10 and -10 euros.

Gain 7 euros.

Option 2

Gain or lose between 20 and -20 euros.

Gain 3 euros.

Introduction

$$\text{Decision } x \rightarrow \text{Observation } \xi \rightarrow \text{Decision } y$$

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- First-stage decision x; (here-and-now)

Challenge: Evaluate consequences of the decision x.

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- First-stage decision x; (here-and-now)
- Uncertain quantity ξ;

Challenge: How to model the uncertainty ξ.

Introduction

Decision x $\rightarrow$ Observation ξ $\rightarrow$ Decision y

- First-stage decision x; (here-and-now)
- Uncertain quantity ξ;
- Second-stage or recourse decision y. (wait-and-see)

Challenge: Recourse decision is a function of uncertainty $y = y(\xi)$.

Motivation

Decision-making over time:

- Generating electrical power using thermal engines or uncertain renewable resources (solar, wind, hydro);
- Planning factory and warehouse installation before knowing demand for products;
- Assigning jobs to people in order to meet a deadline.

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Decision-making over time:

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What is the best decision you can make before all information is available?
How to find that decision?

Outline

The (deterministic) Newsvendor Problem

Suppose you want to buy x newspapers in the morning to sell them along the day.
Each newspaper costs you c, while each sale earns you p.
The number of people interested in buying a newspaper is d.
We assume $0 < c < p$.

Newspapers sold: $\min\{x, d\}$;
Operational costs: $cx - p\min\{x, d\}$;
Operational constraints: $x \geq 0$;

$$\begin{aligned} \min_x \quad & cx - p\min\{x, d\} \\ \text{s.t.} \quad & x \geq 0. \end{aligned}$$

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Now, consider that the demand d is unknown when you buy newspapers.
A possible approach is to treat it as a random variable $\boldsymbol{d}$ with known law.

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How to justify the expectation?
If you repeat this day after day, the Law of Large Numbers guarantees the sample mean (for fixed $x = x$) converges:

$$\lim_{n\to\infty} \frac{\sum_{i=1}^{n} cx - \min\{x, d_i\}}{n} \xrightarrow{\text{a.s.}} \mathbb{E}\left[cx - p\min\{x, \boldsymbol{d}\}\right].$$

The (stochastic) Newsvendor Problem

Often it is possible to make a correction to your initial decision after observing the uncertainty.
Now, assume that each day you can buy an additional y newspapers at noon, when you have certainty of the day's demand.
However, at noon newspapers cost you b, such that $c < b < p$.

$$\begin{aligned} \min_{x,y} \quad & \mathbb{E}\left[cx + by - p\min\{x + y, \boldsymbol{d}\}\right] \\ \text{s.t.} \quad & x \geq 0, \\ & y \geq 0. \end{aligned}$$

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& \boldsymbol{y} \geq 0, \\
& \boldsymbol{y} = \boldsymbol{y}(\boldsymbol{d}).
\end{aligned}
$$

The second decision $\boldsymbol{y}$ *can* depend on the observed demand!
By $\boldsymbol{y} \geq 0$ we mean $\mathbb{P}[\boldsymbol{y} \geq 0] = 1$.

The (stochastic) Newsvendor Problem

Remarks:

- The decision x is not random, thus

$$\mathbb{E}[cx + \cdots] = cx + \mathbb{E}[\cdots];$$

- For each fixed x and d, define

$$\hat{V}(x,d) = \min_{y \geq 0} by - \min\{x + y, d\};$$

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Substituting in our previous formulation, we obtain

$$\begin{aligned} \min_{x} \quad & cx + \mathbb{E}\left[\hat{V}(x, \boldsymbol{d})\right] \\ \text{s.t.} \quad & x \geq 0. \end{aligned}$$

Decomposition

$$\text{Decision } x \to \boxed{\text{Observation } \xi \to \text{ Decision } y}$$

Often, there is a decomposable structure: useful in modeling and algorithms.

Decomposition

$$\overbrace{\text{Decision } x}^{\text{First stage}} \to \overbrace{\boxed{\text{Observation } \xi \to \text{ Decision } y}}^{\text{Second stage}}$$

Often, there is a decomposable structure: useful in modeling and algorithms.

The pieces are called stages. Commonly related to time—but not necessarily.

Thus, we also say that x is the first-stage variable while y is the second-stage variable.

The (stochastic) Newsvendor Problem

First-stage problem:

$$\begin{aligned} \min_{x} \quad & cx + \mathbb{E}\left[\hat{V}(x, \mathbf{d})\right] \\ \text{s.t.} \quad & x \geq 0. \end{aligned}$$

Second-stage problem:

$$\hat{V}(x, d) = \min_{y \geq 0} by - \min\left\{x + y, d\right\};$$

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Solution techniques often employ:

1. Approximation of $\mathbb{E}\left[\hat{V}(x, \boldsymbol{d})\right]$, improved iteratively by...

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Solution techniques often employ:

1. Approximation of $\mathbb{E}\left[\hat{V}(x, \mathbf{d})\right]$, improved iteratively by. . .
2. Monte Carlo sampling of a (tentative) second-stage solution. (Even when solving the second-stage exactly for given x, the first-stage decision can change after approximation improvement.)

The (robust) Newsvendor Problem

An alternative formulation is to assume the demand d belongs to a set D. For example: take $D = \left[\underline{d}, \overline{d}\right]$.

In this setting, we can't take the expectation—d isn't a random variable!

Instead, look at $d \in D$ that results in the worst outcome. This is a *pessimistic* approach.

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First, let us consider the problem without recourse. For a fixed x, the worst outcome is:

$$\max_{d \in D} \Big(-p \min\{x, d\} \Big)$$

The problem then is

$$\begin{aligned} \min_{x} \quad & cx + \max_{d \in D} \Big(-p \min\{x, d\} \Big) \\ \text{s.t.} \quad & x \geq 0. \end{aligned}$$

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With recourse:

$$V(x) = \max_{d \in D} \min_{y \geq 0} \left(by - p \min\left\{x + y, d\right\} \right)$$

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When D is infinite, as in our case $D = \left[\underline{d}, \overline{d}\right]$, this results in a *semi-infinite problem*!

The (robust) Newsvendor Problem

Solving a robust optimization problem often requires a reformulation to merge min and max problems and represent an infinite number of constraints in a tractable manner.
It can also involve an approximation of V instead of or in addition to.

Comparison

Robust optimization can be more resilient to bad outcomes. Also, it can employ a simpler uncertainty model.
However, in comparison with stochastic optimization, it can overestimate the cost.

Outline

The (two-stage) Newsvendor Problem

Consider that your true problem isn't to buy-and-sell newspapers in a single day, but rather over multiple days.
For example, assume that you consider the stochastic problem over two days.

$$\underset{\hookrightarrow x_1}{\text{Decision}} \rightarrow \overset{\rightsquigarrow d_1}{\text{Observation}} \rightarrow \underset{\hookrightarrow y_1, x_2}{\text{Decision}} \rightarrow \overset{\rightsquigarrow d_2}{\text{Observation}} \rightarrow \underset{\hookrightarrow y_2}{\text{Decision}}$$

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But why remember the demand $\boldsymbol{d}_1$ when choosing how many newspapers to buy $\boldsymbol{x}_2$?
Past events can influence future (uncertain) events! (And our *prediction* of future events.)

Multistage problems

More generally, we can consider multistage problems with an arbitrary number of stages.

$$\text{Decision} \rightarrow \text{Observation} \rightarrow \text{Decision} \rightarrow \text{Observation} \rightarrow \text{Decision} \rightarrow \text{Observation} \rightarrow \ldots \rightarrow \text{Observation} \rightarrow \text{Decision}$$

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We call a full sequence of observations a scenario. Each scenario has a corresponding probability of occurring (prob. density if continuous).

Multistage problems

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Let us look at what happens if we try to solve the problem directly.

Exploiting finiteness, we can construct a *deterministic* optimization problem equivalent to the original stochastic problem.
This is done by adding a decision variable for each scenario.

The (two-stage) Newsvendor Problem (revisited)

For example:

$$\begin{aligned} \min_{x,y} \quad & \sum_{s=1}^{S} p^s \Big(c x_1^s + b y_1^s - p \min\{x_1^s + y_1^s, d_1^s\} + \\ & \qquad c x_2^s + b y_2^s - p \min\{x_2^s + y_2^s, d_2^s\} \Big) \\ \text{s.t.} \quad & x \geq 0, y \geq 0. \end{aligned}$$

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Is that all?

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$$\text{s.t.} \quad x \geq 0, y \geq 0.$$

Is that all? No!

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These are nonanticipativity constraints.

(In particular, note that all x_1^s must be equal, because that decision is taken before any random variable is observed.)

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Main aspects to consider.

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- How to model uncertainty?
- How to represent your attitude toward risk?
- What information structure to choose?

Questions?

Bibliography I

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