Optimal Hospital Care Scheduling During the SARS-CoV-2 Pandemic

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Daily COVID Cases Remain Stubbornly High in the UK

Source: GOV.UK
A&E patients are waiting up to 13 HOURS for a bed as health bosses warn health service faces toughest winter ever — but NHS England boss says it needs more staff not cash

- Doctors in Newcastle said ‘become very normal’ to wait 7+ hours for A&E bed
- NHS ‘facing toughest winter ever’ due to staff shortages, pandemic backlogs
- Amanda Pritchard - NHS England boss - suggested throwing money won't fix it

NHS is 'on its knees' even WITHOUT a Covid surge: Heart attack and stroke sufferers face 55 MINUTE ambulance waits with patients 'dying in waiting rooms' because they can't be seen - as backlog for routine treatment in England hits record 5.83million

By John Ely Senior Health Reporter For Mailonline and Emily Craig Health Reporter For Mailonline
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A Silent Killer in the Wait

Size of the waitlist for elective treatment

Source: Edge Health
Not Only are More Waiting, But They are also Waiting for Longer

Huge rise in waits of more than a year
Patients waiting more than 52 weeks for routine treatments in England, by month

Source: NHS England
How to Prioritize COVID vs Non-COVID Patients?

Opinion UK politics & policy

Agonising choices in ICUs should be made by society, not individuals

Covid is overwhelming hospitals and forcing clinicians to decide who does and who does not receive care

Clinical staff in an intensive care unit. There is a shortage of about 40,000 nurses in the NHS. This has affected the Covid response © Neil Hall/EPA/Shutterstock
1. Weakly Coupled Counting Dynamic Programs
2. The Fluid Approximation
3. Performance Guarantees
4. Case Study
We model each individual patient as a dynamic program:

**Dynamic Program**

- finite time stages $\mathcal{T}$, states $\mathcal{S}$, actions $\mathcal{A}$
- initial probabilities $q(s)$
- Markovian transition probabilities $p_t(s'|s,a)$
- expected rewards $r_t(s,a)$

**Objective**

Find policy $\pi = \{\pi_t\}_{t \in \mathcal{T}}$ with $\pi_t : \mathcal{S} \rightarrow \mathcal{A}$ that maximizes the expected total rewards $\mathbb{E} \left[ \sum_{t \in \mathcal{T}} r_t(\tilde{s}_t, \pi_t(\tilde{s}_t)) \right]$
Dynamic Programs (DPs)

DP model for an individual patient:
Dynamic Programs (DPs)
Dynamic Programs (DPs)

Time:

Action: $\emptyset$
Dynamic Programs (DPs)

Time:
Dynamic Programs (DPs)

Time:

Action: $\emptyset$

[Diagram showing various actions and time progression with symbols and time labels such as DEC 3 and Skull symbol.]

3
Dynamic Programs (DPs)

Time: 

Action: 

3
Dynamic Programs (DPs)

Time:

\[ \emptyset \]

\[ \emptyset \]
Dynamic Programs (DPs)

Time:

Action:
Dynamic Programs (DPs)

Time:

GA

GA

CC

CC

CC

Time: 3
Dynamic Programs (DPs)

Time:

Action: ∞
Dynamic Programs (DPs)

Time:

Action: GA

[Diagram showing a time line with actions and transitions labeled as GA and CC]
Dynamic Programs (DPs)

Time:

- GA
- CC
- ∅
Dynamic Programs (DPs)

Time:

Action: ∅
Dynamic Programs (DPs): Extensive Model

Can we combine all patient DPs to one large DP of the overall health system?

The state and action spaces satisfy

\[ |\mathcal{S}| = \prod_{i \in \mathcal{I}} |\mathcal{S}_i| \quad \text{and} \quad |\mathcal{A}| = \prod_{i \in \mathcal{I}} |\mathcal{A}_i| \]

In our case: 10m patients \( \times \) 15 states, 6 actions,

\[ |\mathcal{S}| = 15^{10,000,000} \quad \text{and} \quad |\mathcal{A}| = 6^{10,000,000} \]

Any policy needs to map

\[ |\mathcal{S}| = 15^{10,000,000} \quad \text{to} \quad |\mathcal{A}| = 6^{10,000,000} \]
Multiple patients of the same group can be modelled by a counting DP:

For $n$ iid DPs $(\mathcal{S}, \mathcal{A}, q, p, r)$ over same $\mathcal{T}$, a DP $(\mathcal{G}, \mathcal{U}, q, p, r)$ with:

- **states** $\mathcal{G} = \{\sigma : \mathcal{S} \rightarrow \mathbb{N}_0\}$, **actions** $\mathcal{U} = \{\alpha : \mathcal{S} \times \mathcal{A} \rightarrow \mathbb{N}_0\}$
- **initial prob’s** $q(\sigma) = \frac{n!}{\prod_{s \in \mathcal{S}} \sigma(s)!} \cdot \prod_{s \in \mathcal{S}} q(s)^{\sigma(s)}$
- **transition prob’s**
  $$p_t(\sigma' | \sigma, \alpha) = \sum_{\theta \in \Gamma(\sigma, \alpha, \sigma')} \prod_{s \in \mathcal{S}} \prod_{a \in \mathcal{A}} \left[ \frac{\alpha(s, a)!}{\prod_{s' \in \mathcal{S}} \theta(s, a, s')!} \cdot \prod_{s' \in \mathcal{S}} p_t(s' | s, a)^{\theta(s, a, s')} \right]$$
- **rewards** $r_t(\sigma, \alpha) = \sum_{s \in \mathcal{S}} \sum_{a \in \mathcal{A}} r_t(s, a) \cdot \alpha(s, a)$
Multiple patients of the same group can be modelled by a counting DP:

For \( n \) iid DPs \((\mathcal{S}, \mathcal{A}, q, p, r)\) over same \( \mathcal{T} \), a DP

- states \( \mathcal{G} = \{\sigma : \mathcal{S} \to \mathbb{N}_0\} \), actions \( \mathcal{A} = \{\alpha : \mathcal{S} \times \mathcal{A} \to \mathbb{N}_0\} \)
- initial prob’s \( q(\sigma) = \frac{n!}{\prod_{s \in \mathcal{S}} \sigma(s)! \cdot \prod_{s \in \mathcal{S}} q(s)^{\sigma(s)}} \)
- transition prob’s
  \[
  \psi_t(\sigma' | \sigma, \alpha) = \sum_{\theta \in \Gamma(\sigma, \alpha, \sigma')} \prod_{s \in \mathcal{S}} \prod_{a \in \mathcal{A}} \left[ \frac{\alpha(s, a)!}{\prod_{s' \in \mathcal{S}} \theta(s, a, s')!} \right]
  \]
- rewards \( r_t(\sigma, \alpha) = \sum_{s \in \mathcal{S}} \sum_{a \in \mathcal{A}} r_t(s, a) \cdot \alpha(s, a) \)
Multiple patients of the same group can be modelled by a counting DP:

For \( n \) iid DPs \( (\mathcal{S}, \mathcal{A}, q, p, r) \) over same \( \mathcal{T} \), a DP \( (\mathcal{G}, \mathcal{A}, q, p, r) \) with:

- states \( \mathcal{G} = \{ \sigma : \mathcal{S} \to \mathbb{N}_0 \} \), actions \( \mathcal{A} = \{ \alpha : \mathcal{S} \times \mathcal{A} \to \mathbb{N}_0 \} \)
- initial prob’s \( q(\sigma) = \frac{n!}{\prod_{s \in \mathcal{S}} \sigma(s)} \)
- multinomial \( (\alpha(s, a); p_t(\cdot | s, a)) \)
- transition prob’s

\[
\psi_t(\sigma' | \sigma, \alpha) = \sum_{\theta \in \Gamma(\sigma, \alpha, \sigma')} \prod_{s \in \mathcal{S}} \prod_{a \in \mathcal{A}} \frac{\alpha(s, a)!}{\prod_{s' \in \mathcal{S}} \theta(s, a, s')!} \cdot \prod_{s' \in \mathcal{S}} p_t(s' | s, a)^{\theta(s, a, s')} 
\]

- rewards \( r_t(\sigma, \alpha) = \sum_{s \in \mathcal{S}} \sum_{a \in \mathcal{A}} r_t(s, a) \cdot \alpha(s, a) \)
Counting DPs

Time:

3,200

- Race flag
- December 3 calendar
- Ambulance
- Hospital bed
- Skull and crossbones
Counting DPs

Time:

3,200 x 6

3,200
Time: 3,200
Counting DPs

Time: 2,562

638
Counting DPs

Time: [Diagram with bars]

2,562 ⇒ 168 x GA
2,394 ⇒ 638 x CC

638 ⇒ 638 x CC
Counting DPs

Time:

- 2,310
- 168
- 638
- 84
Counting DPs

Time:

2,310 $\Rightarrow$ 132 x GA
2,178 $\Rightarrow$ 168 x GA
168 $\Rightarrow$ 168 x GA
638 $\Rightarrow$ 638 x CC
84 $\Rightarrow$ 64 x CC
20 $\Rightarrow$ 20 x CC
Counting DPs

Time:

- 2,178
- 276
- 698
- 24
- 4
- 20
Counting DPs achieve exponential compression over individual DPs:
For \( n \) patient DPs with \( S \) states each, the state spaces scale according to

\[
| \mathcal{S} | = S^n \\
| \mathcal{G} | \approx n^S
\]

- Extensive Model
- Counting DP

\[
\begin{align*}
| \mathcal{S} | &= S^n \\
| \mathcal{G} | &\approx n^S
\end{align*}
\]

\[
\begin{align*}
100\% &\quad \text{compression} \\
3.17 \cdot 10^{-85}\% &\quad \text{compression}
\end{align*}
\]

- \( n = 15 \)
- \( n = 100 \)
Can we combine all patient DPs to one large counting DP?

The state and action spaces satisfy

\[ |\mathcal{S}| \approx |\mathcal{I}|^{\sigma_i} \quad \text{and} \quad |\mathcal{A}| \approx |\mathcal{I}|^{\sigma_i \cdot |\mathcal{A}_i|} \]

In our case: 10m patients à 15 states, 6 actions,

\[ |\mathcal{S}| \approx 10,000,000^{15} \quad \text{and} \quad |\mathcal{A}| \approx 10,000,000^{15 \cdot 6} \]

Any policy needs to map

\[ |\mathcal{S}| \approx 10,000,000^{15} \quad \text{to} \quad |\mathcal{A}| \approx 10,000,000^{15 \cdot 6} \]
Multiple patients groups can be modelled by a weakly coupled counting DP:

For the $J$ counting DPs $(\mathcal{S}_j, \mathcal{A}_j, q_j, p_j, r_j)$ with $n_j$ iid DPs each, over same $\mathcal{T}$, a DP $(\mathcal{S}, \mathcal{A}, q, p, r)$ with:

- states $\mathcal{S} = \times_j \mathcal{S}_j$, actions $\mathcal{A} = \times_j \mathcal{A}_j$
- resource constraints $\sum_j \sum_s \sum_a c_{ilj}(s, a) \cdot \alpha_j(s, a) \leq b_{tl} \quad \forall l \in \mathcal{L}, \forall t \in \mathcal{T}$
- initial prob’s $q(\sigma) = \prod_j q_j(\sigma_j)$, transitions $p_t(\sigma' | \sigma, \alpha) = \prod_j p_{jt}(\sigma'_j | \sigma_j, \alpha_j)$
- rewards $r_t(\sigma, \alpha) = \sum_j r_{jt}(\sigma_j, \alpha_j)$