# Optimal Hospital Care Scheduling During the SARS-CoV-2 Pandemic

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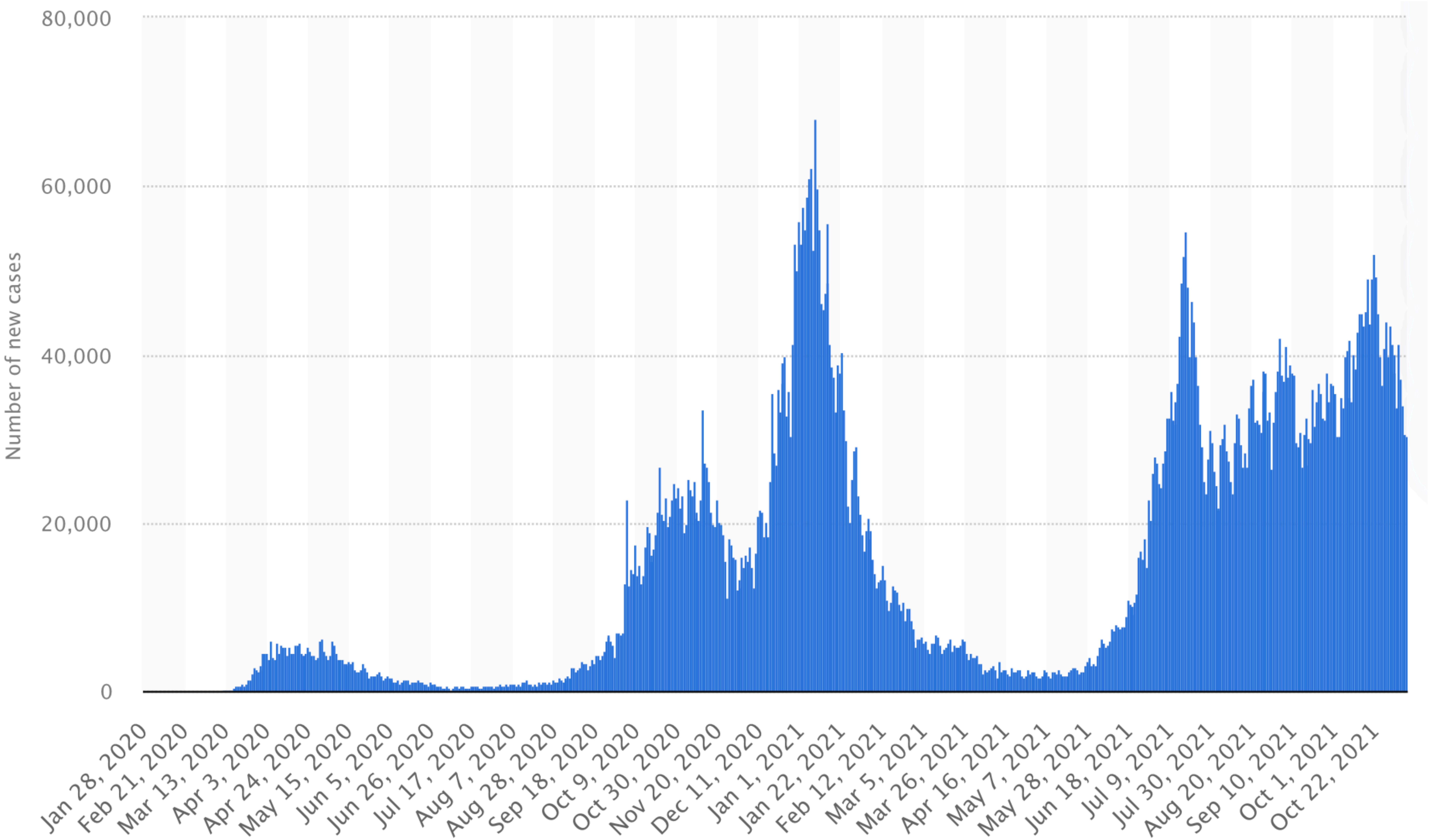
# Daily COVID Cases Remain Stubbornly High in the UK




**Source:** GOV.UK

# The Fallout:
# A Hopelessly Overstretched NHS


**A&E patients are waiting up to 13 HOURS for a bed as health bosses warn health service faces toughest winter ever — but NHS England boss says it needs more staff not cash**

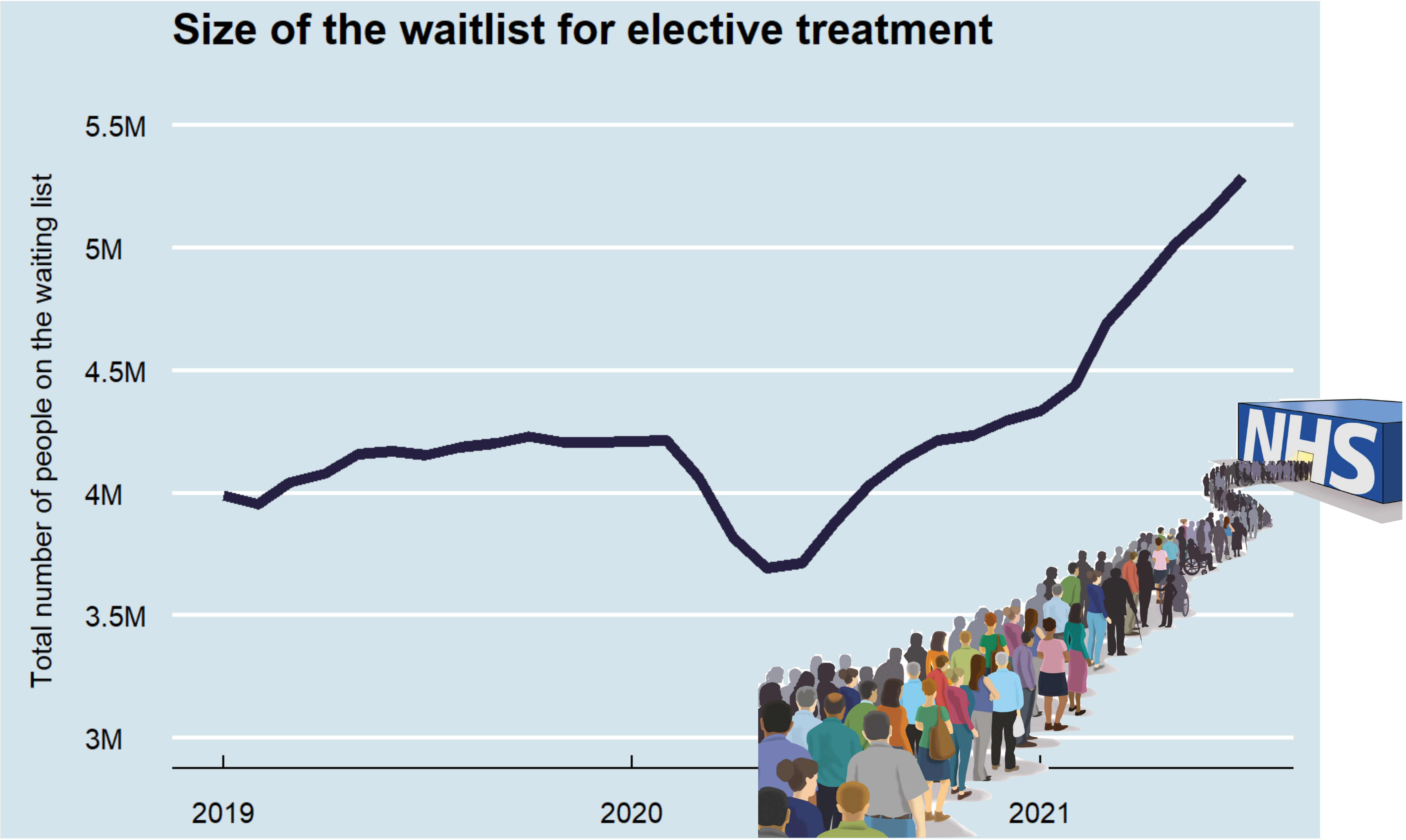
- **Doctors in Newcastle said 'become very normal' to wait 7+ hours for A&E bed**
- **NHS 'facing toughest winter ever' due to staff shortages, pandemic backlogs**
- **Amanda Pritchard - NHS England boss - suggested throwing money won't fix it**

By CONNOR BOYD DEPUTY HEALTH EDITOR FOR MAILONLINE
**PUBLISHED:** 10:01, 16 November 2021 | **UPDATED:** 10:32, 16 November 2021

NHS is 'on its knees' even WITHOUT a Covid surge: Heart attack and stroke sufferers face 55 MINUTE ambulance waits with patients 'dying in waiting rooms' because they can't be seen - as backlog for routine treatment in England hits record 5.83million

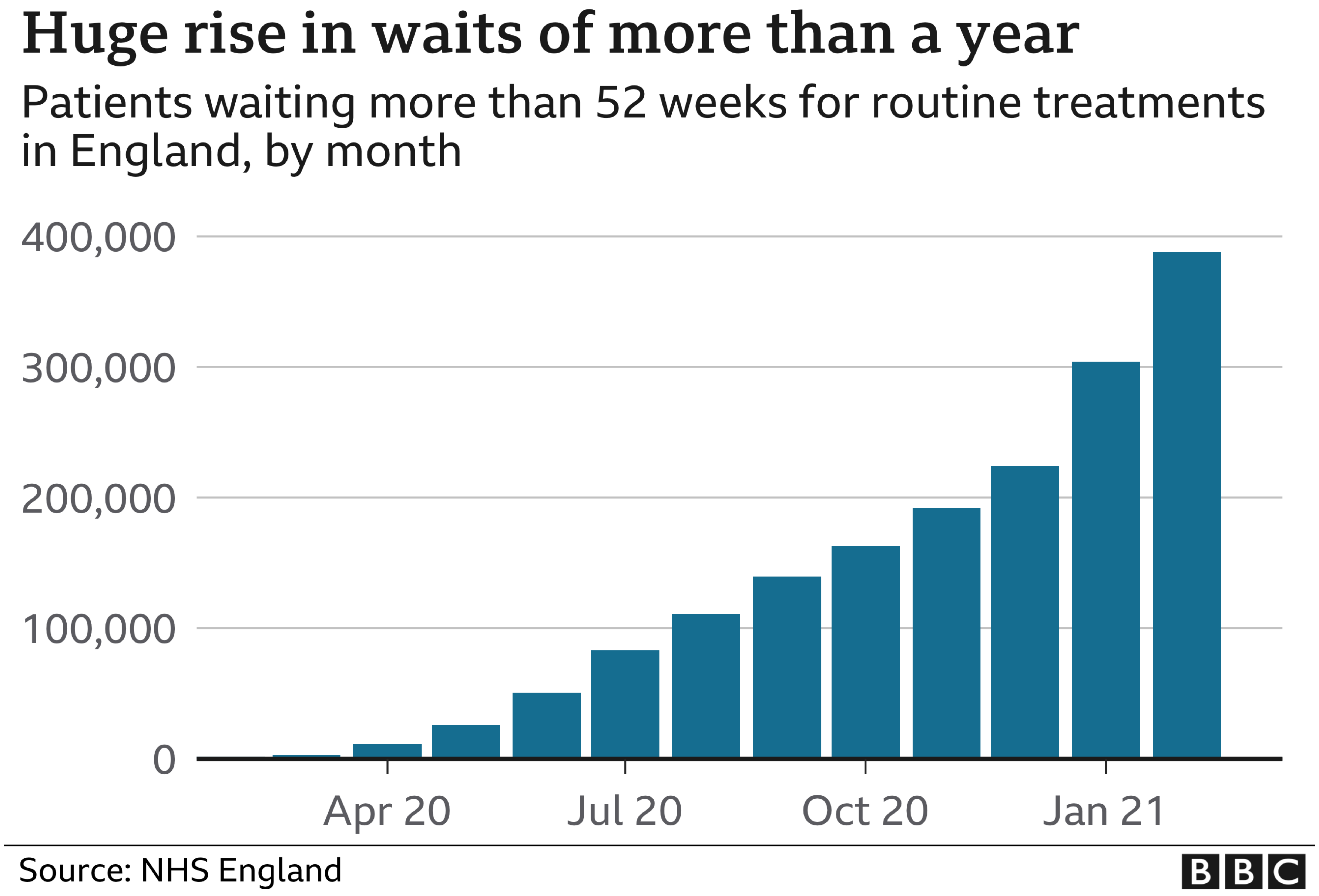
By John Ely Senior Health Reporter For Mailonline and Emily Craig Health Reporter For Mailonline
07:38, 11 Nov 2021 , updated 19:32, 11 Nov 2021

# A Silent Killer in the Wait




**Source:** Edge Health

# Not Only are More Waiting, But They are also Waiting for Longer



# How to Prioritize COVID vs Non-COVID Patients?

Opinion **UK politics & policy**

## Agonising choices in ICUs should be made by society, not individuals

Covid is overwhelming hospitals and forcing clinicians to decide who does and who does not receive care

Clinical staff in an intensive care unit. There is a shortage of about 40,000 nurses in the NHS. This has affected the Covid response

**We model each individual patient as a dynamic program:**

**Dynamic Program**

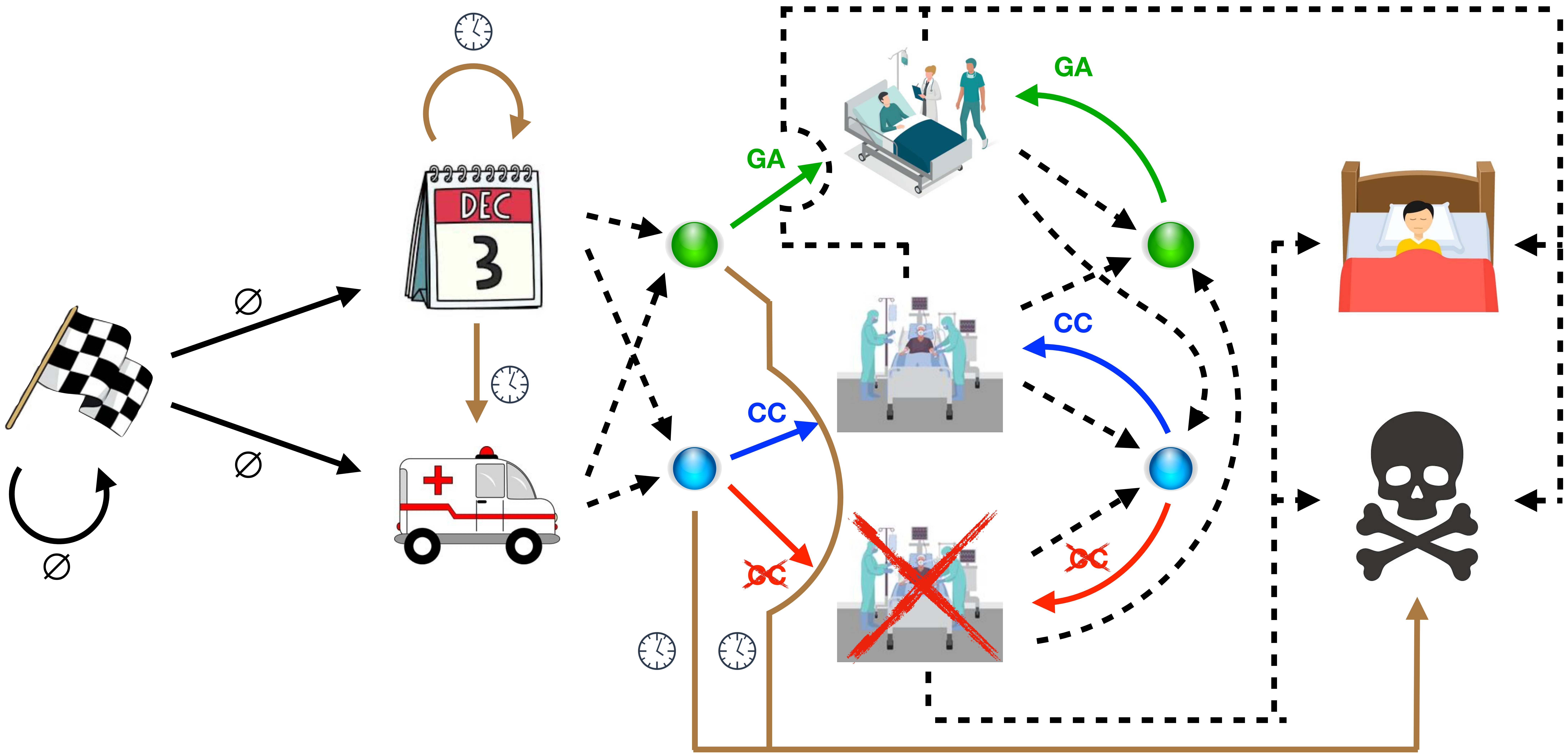
- finite time stages $\mathcal{T}$, states $\mathcal{S}$, actions $\mathcal{A}$
- initial probabilities $q(s)$
- Markovian transition probabilities $p_t(s' \mid s, a)$
- expected rewards $r_t(s, a)$

**Objective**

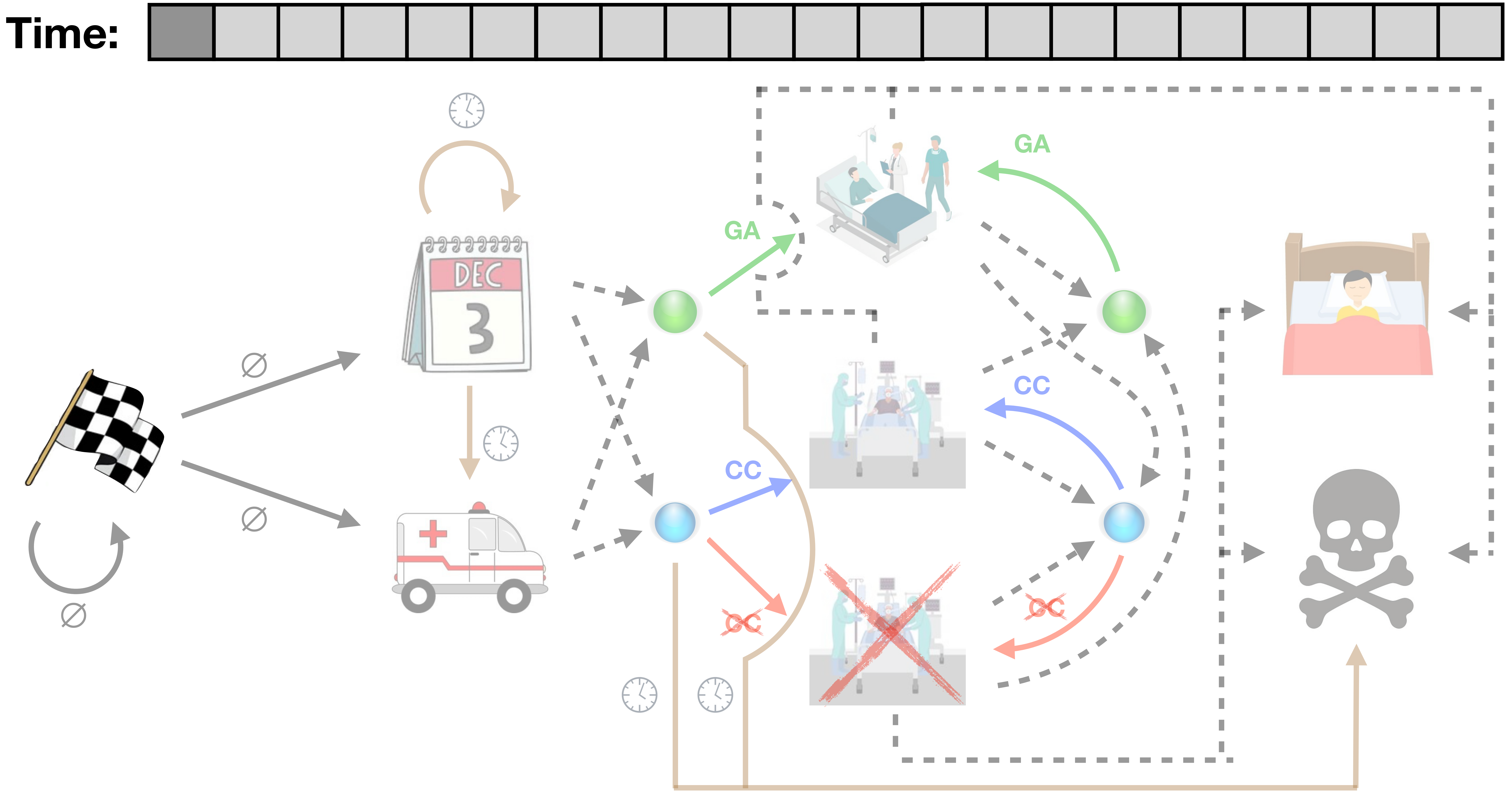
Find policy $\pi = \{\pi_t\}_{t \in \mathcal{T}}$ with $\pi_t : \mathcal{S} \to \mathcal{A}$ that

maximizes the expected total rewards $\mathbb{E}\left[\sum_{t \in \mathcal{T}} r_t(\tilde{s}_t, \pi_t(\tilde{s}_t))\right]$

**DP model for an individual patient:**



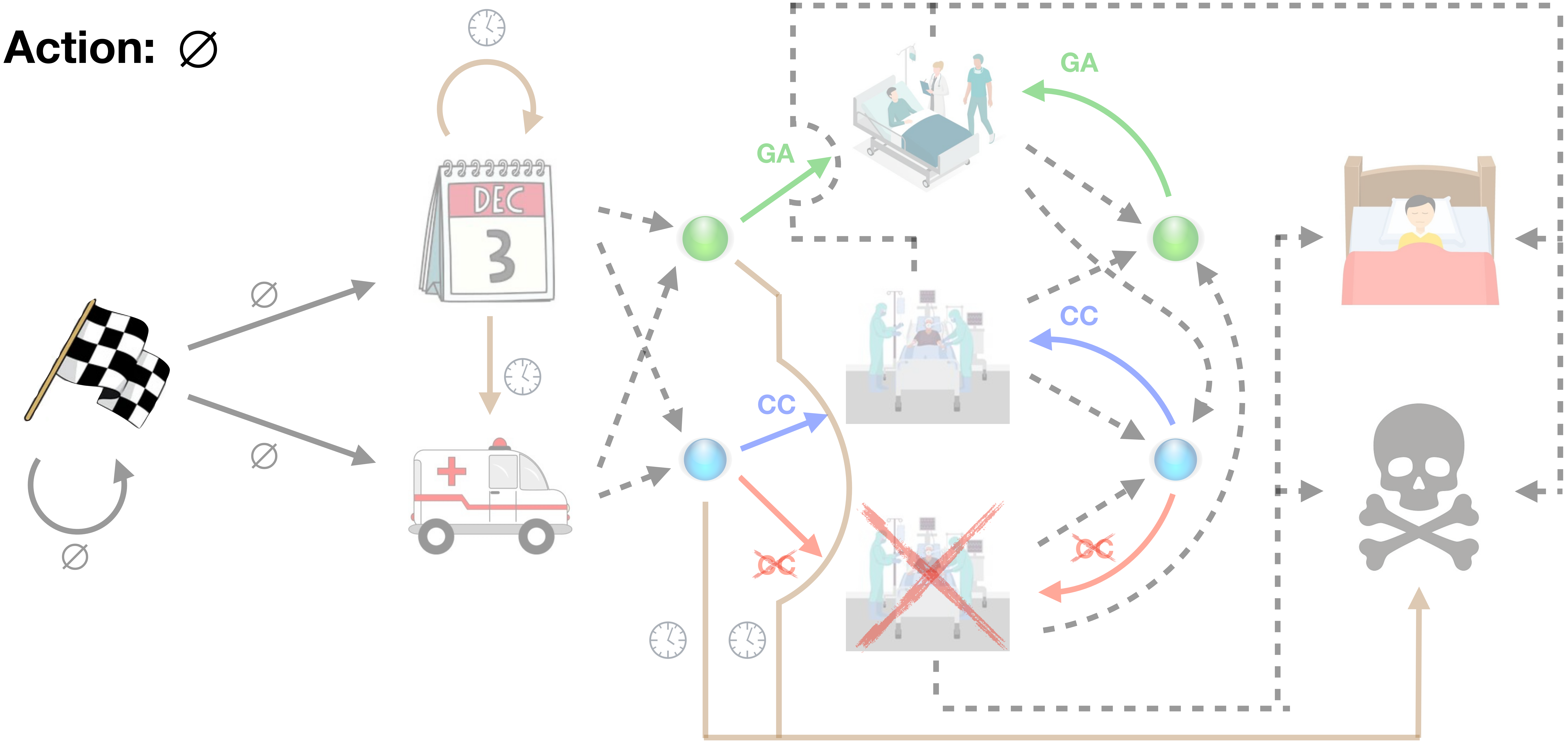
# Dynamic Programs (DPs)



# Dynamic Programs (DPs)

**Time:**

**Action:** ∅



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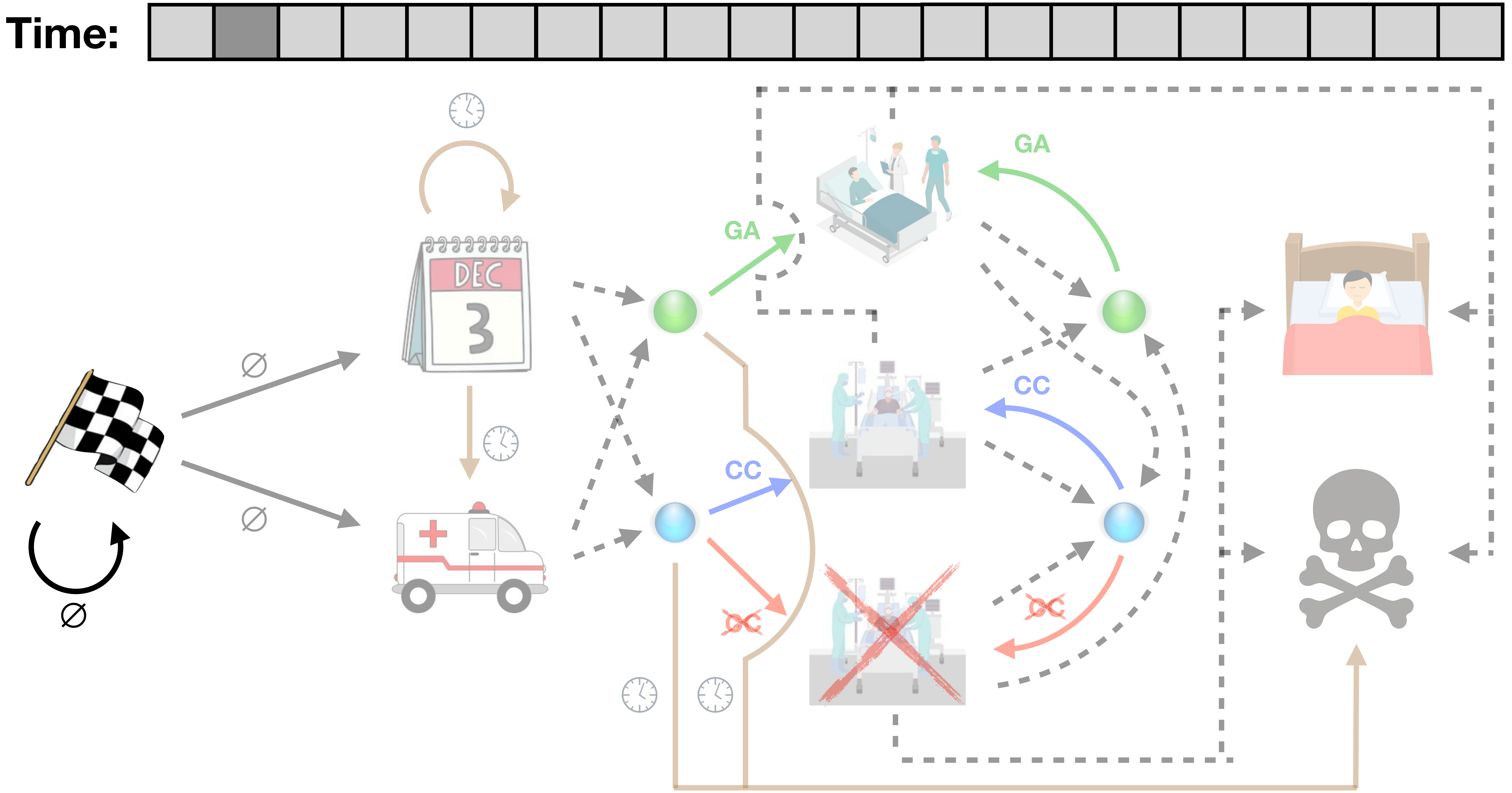

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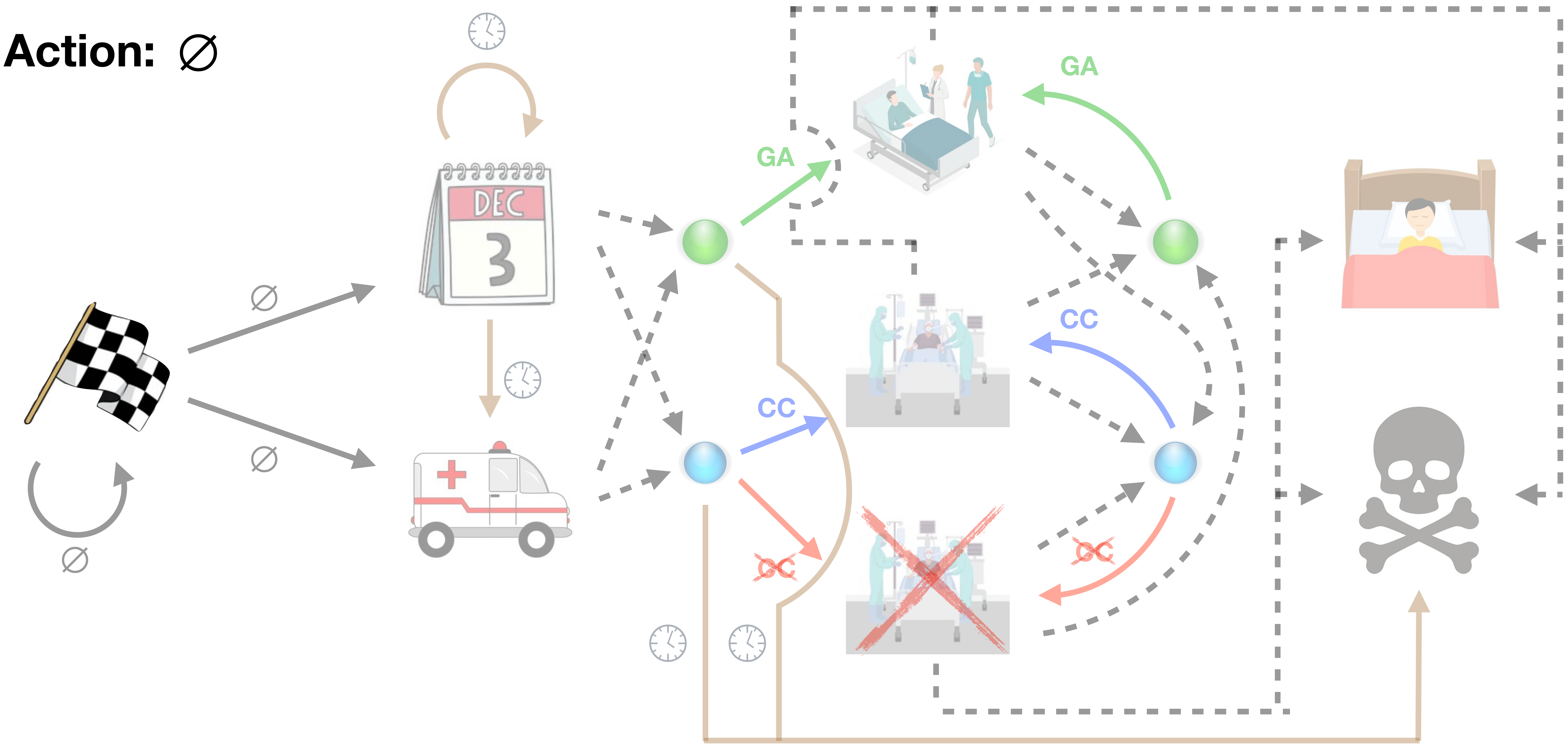

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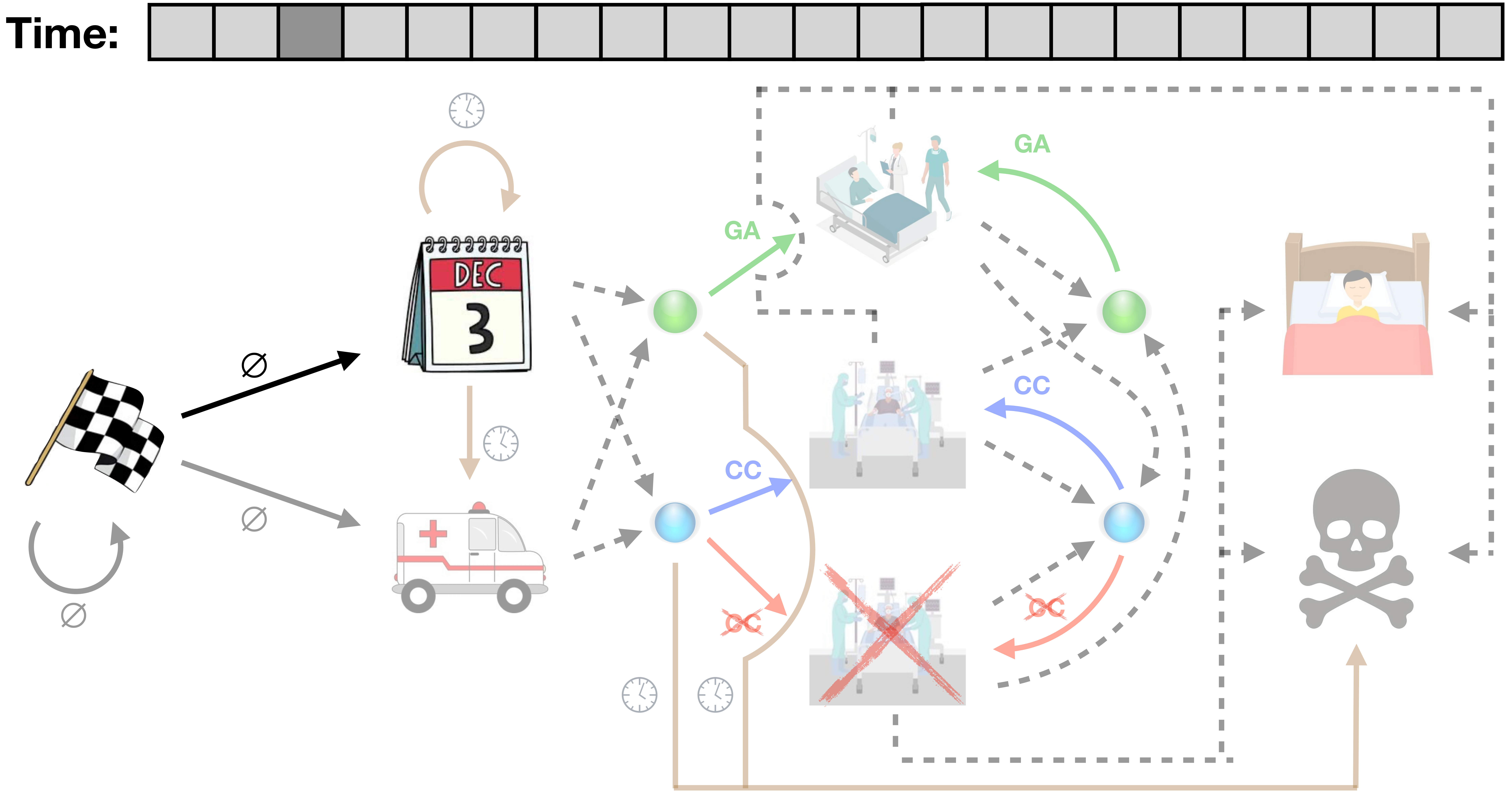

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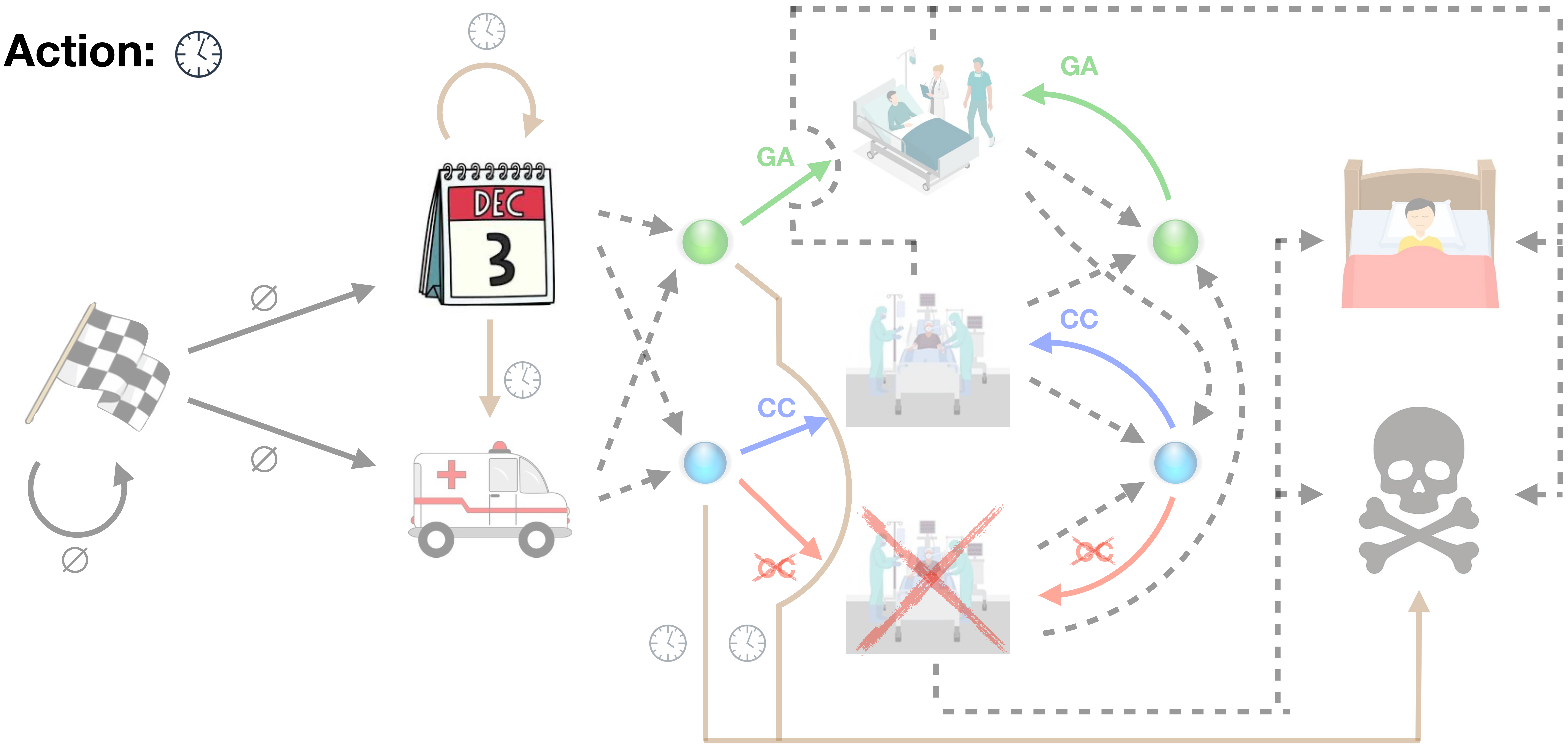

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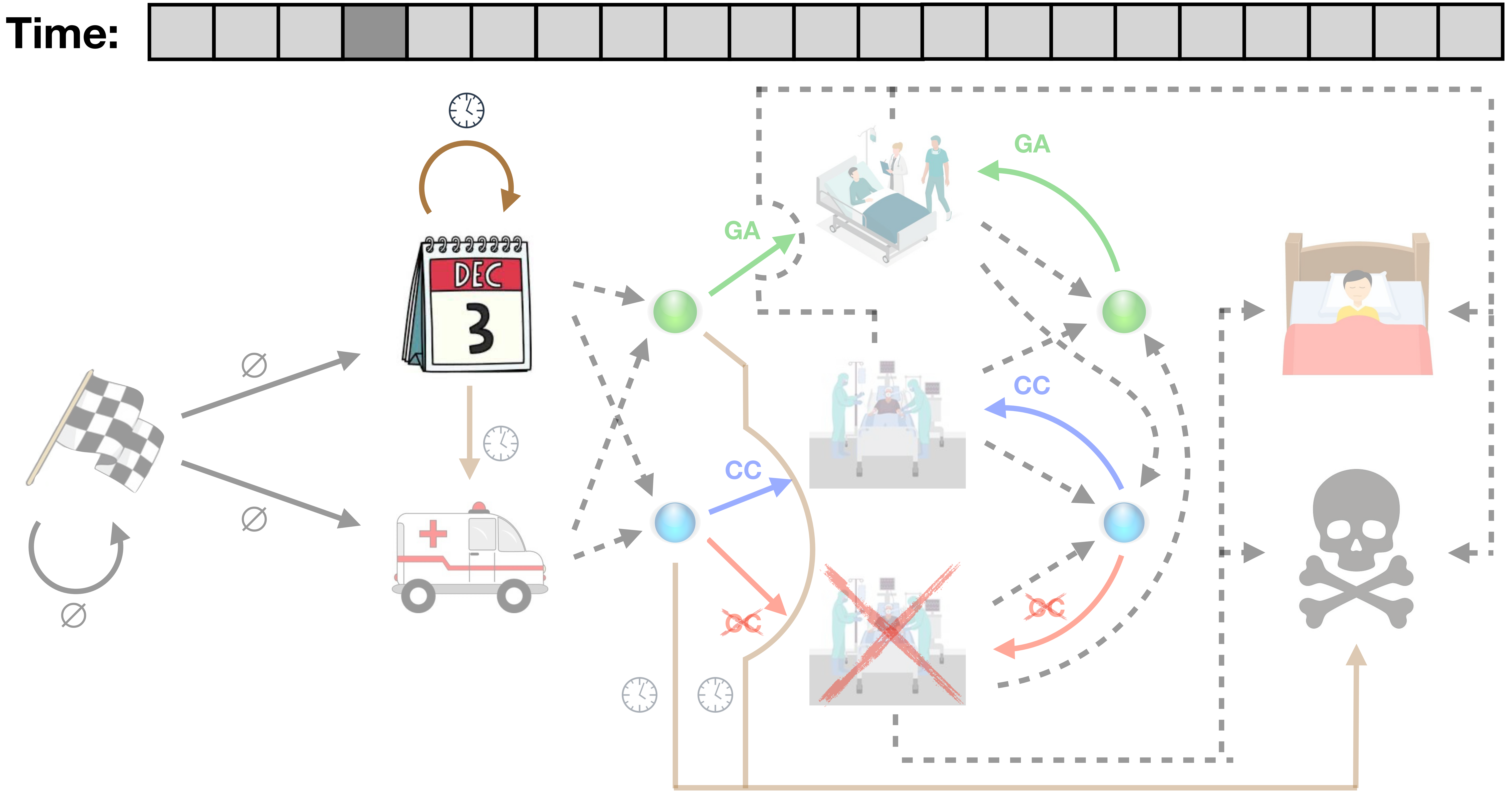

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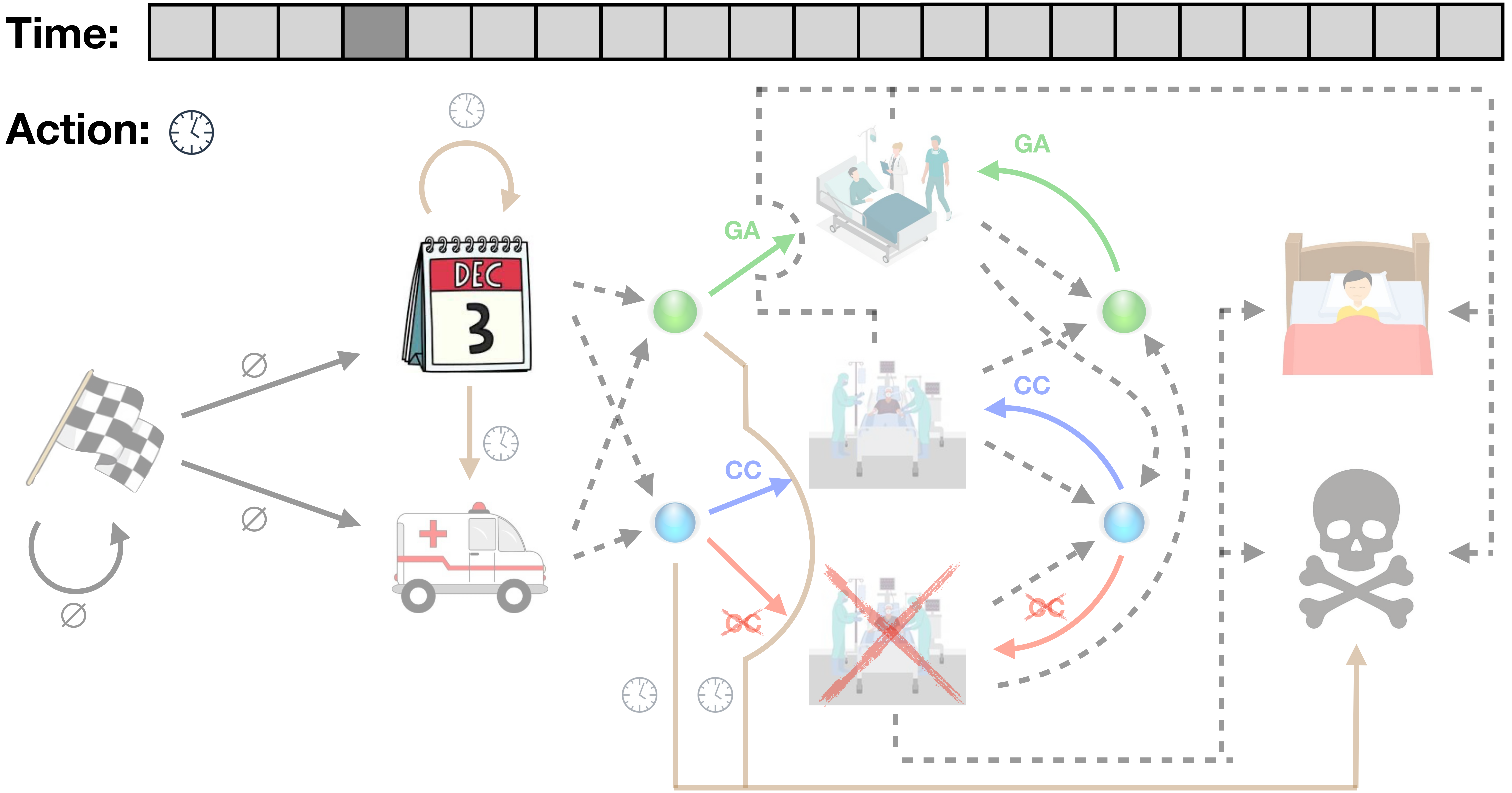

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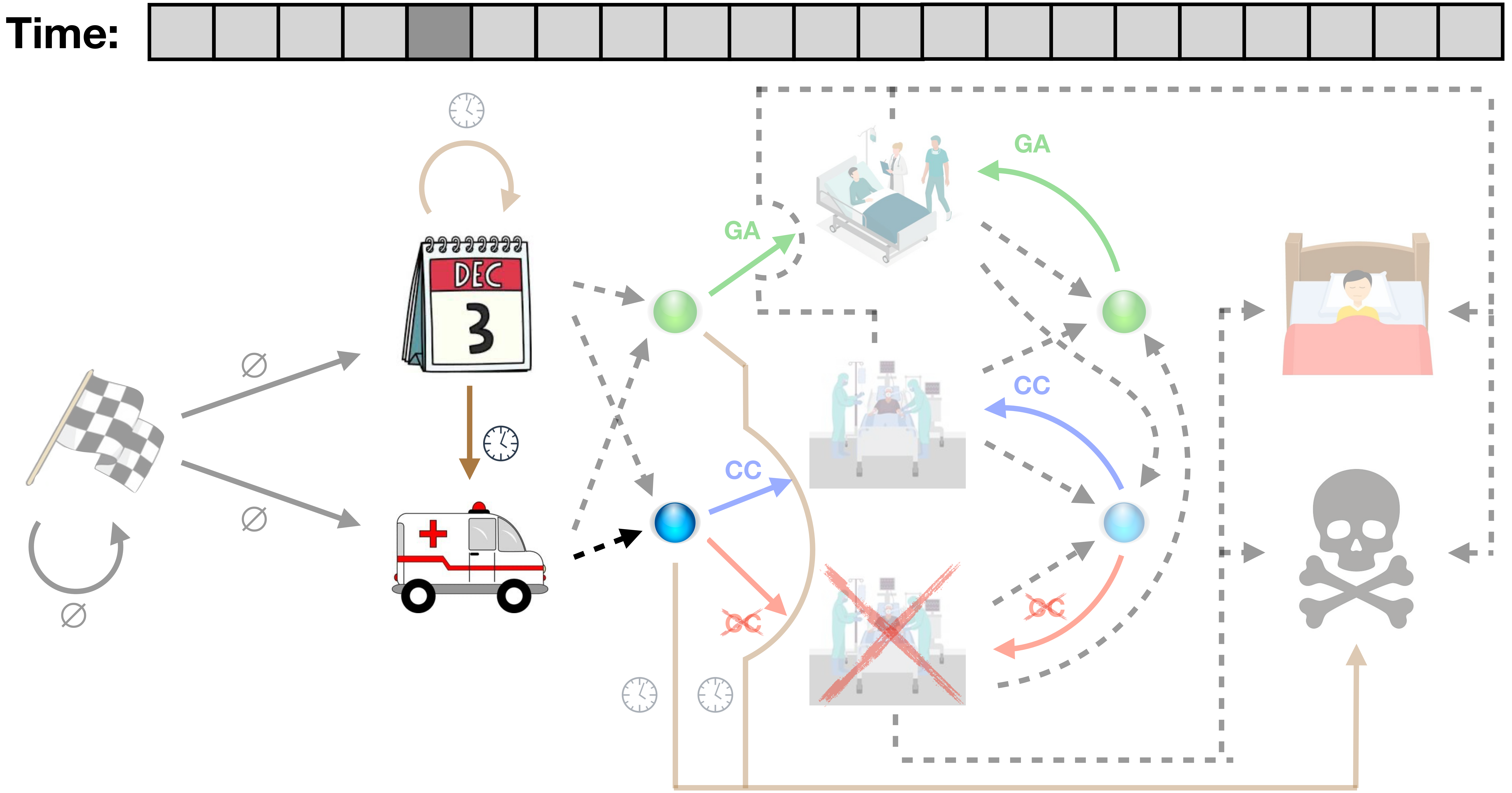

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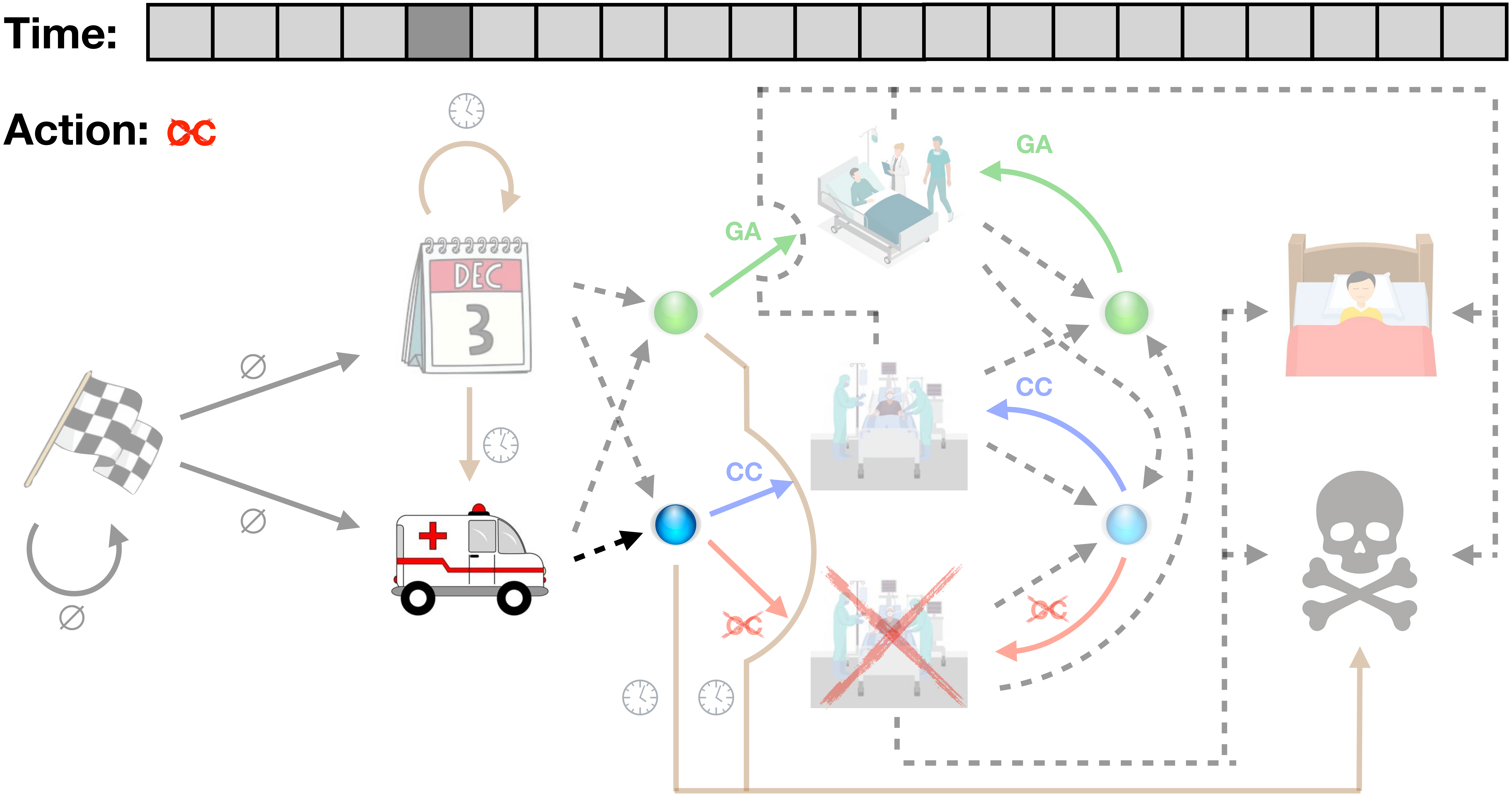

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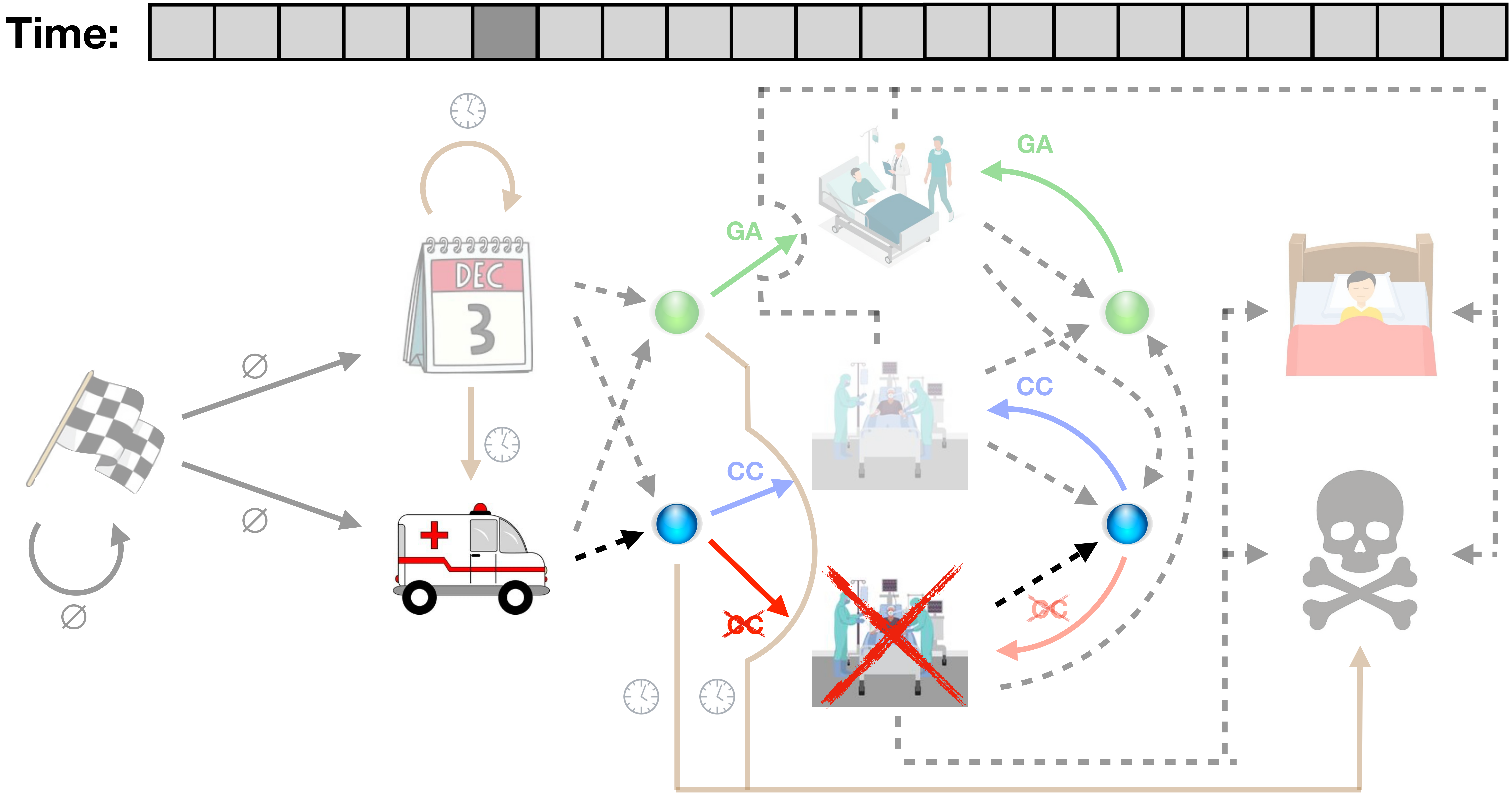

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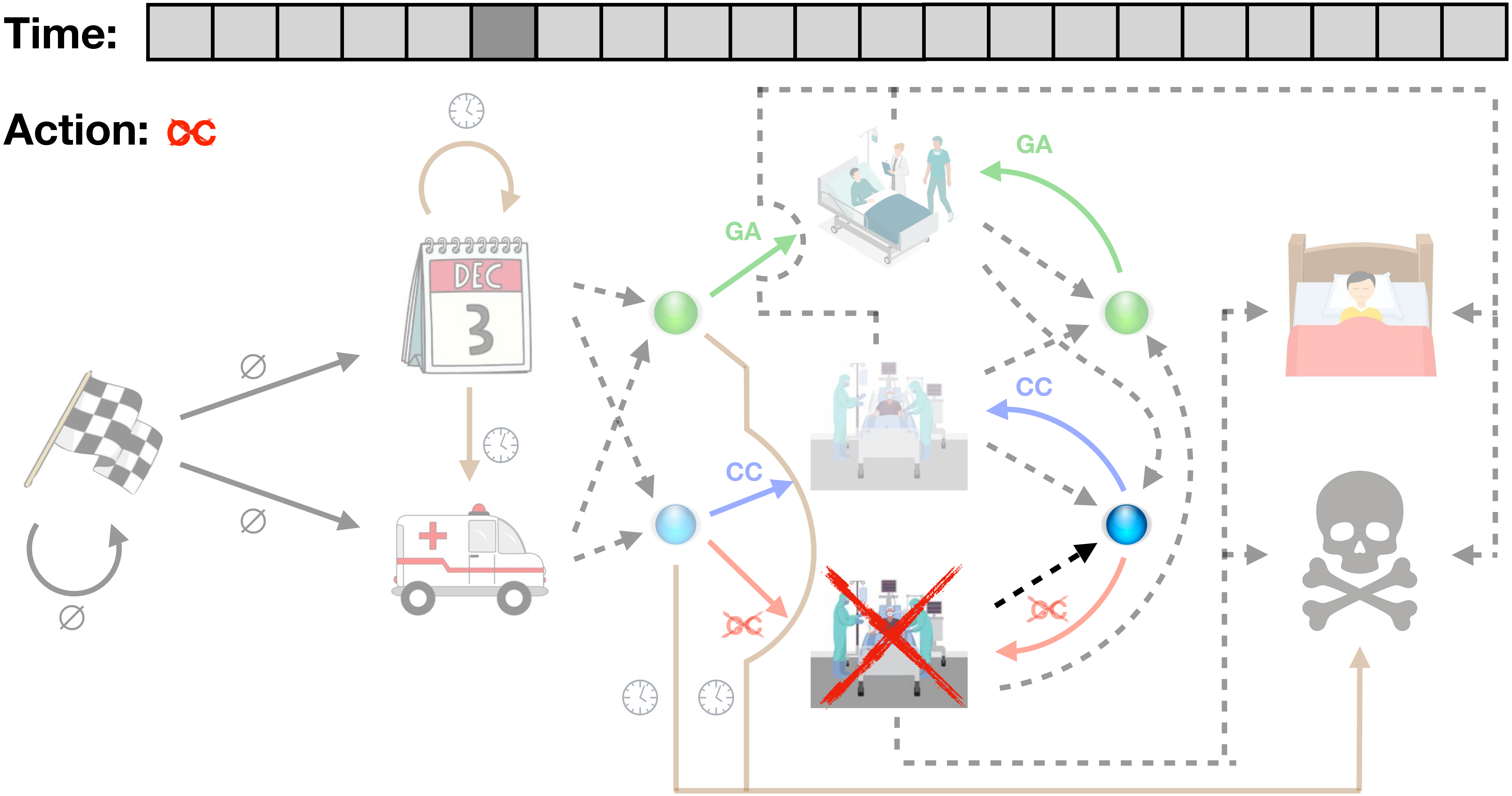

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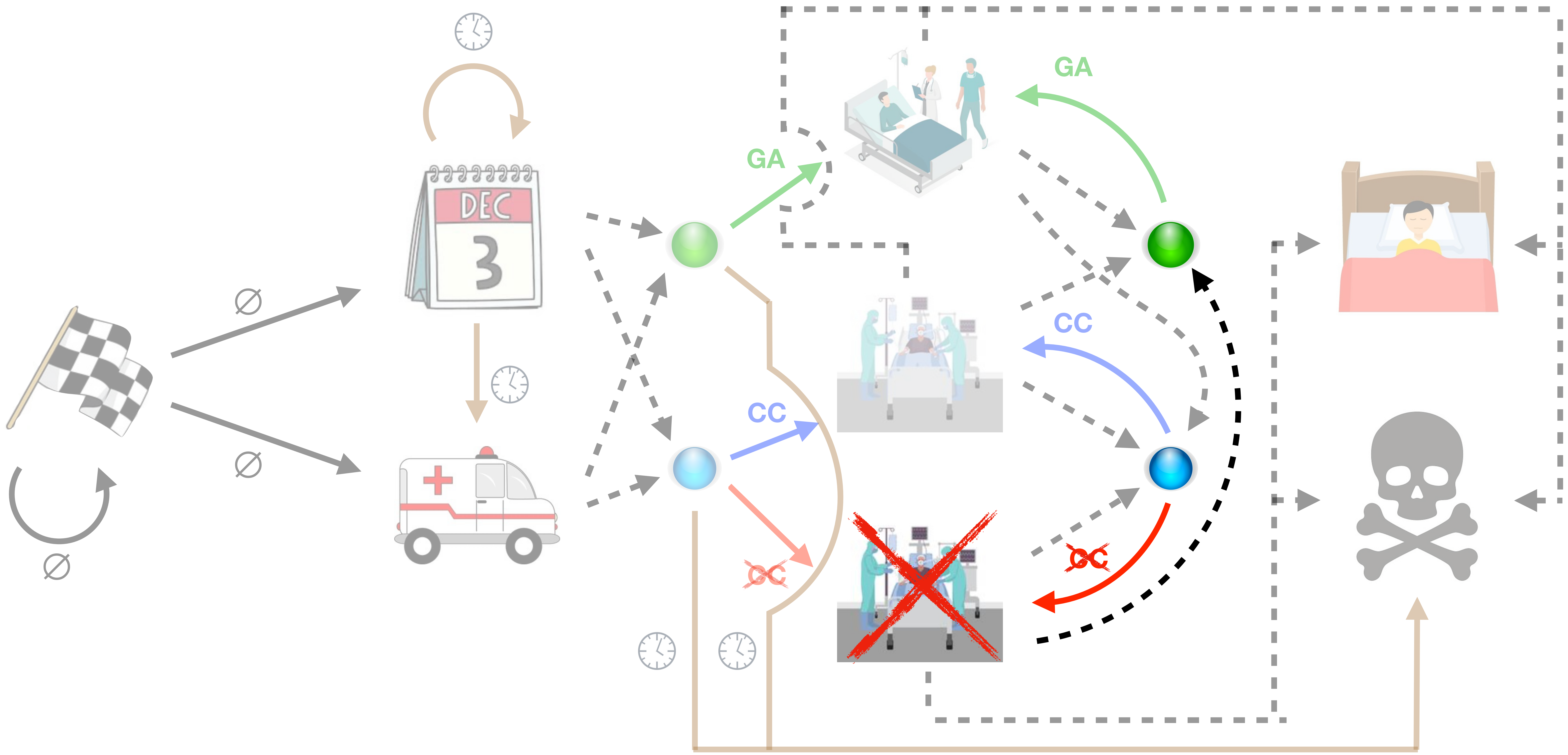

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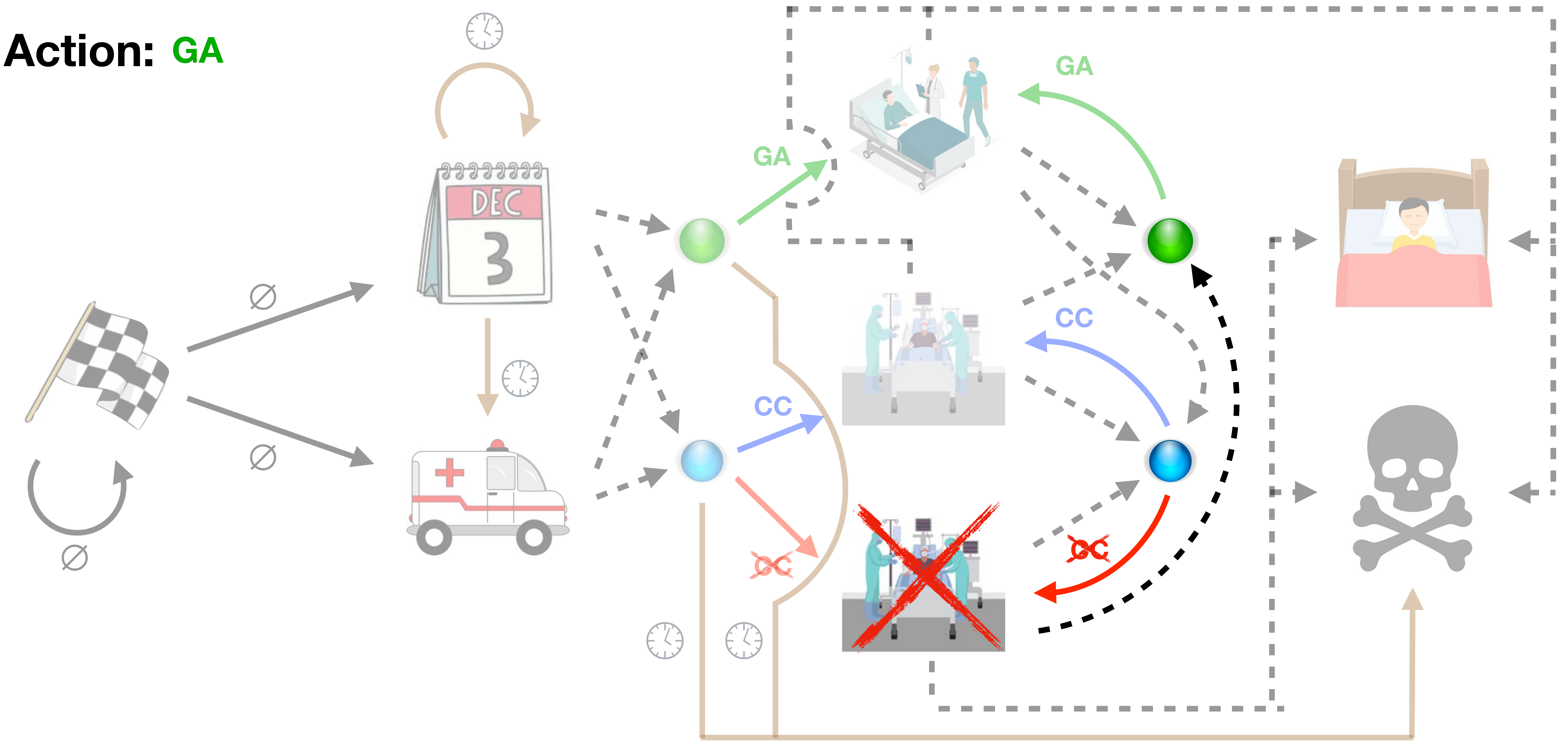

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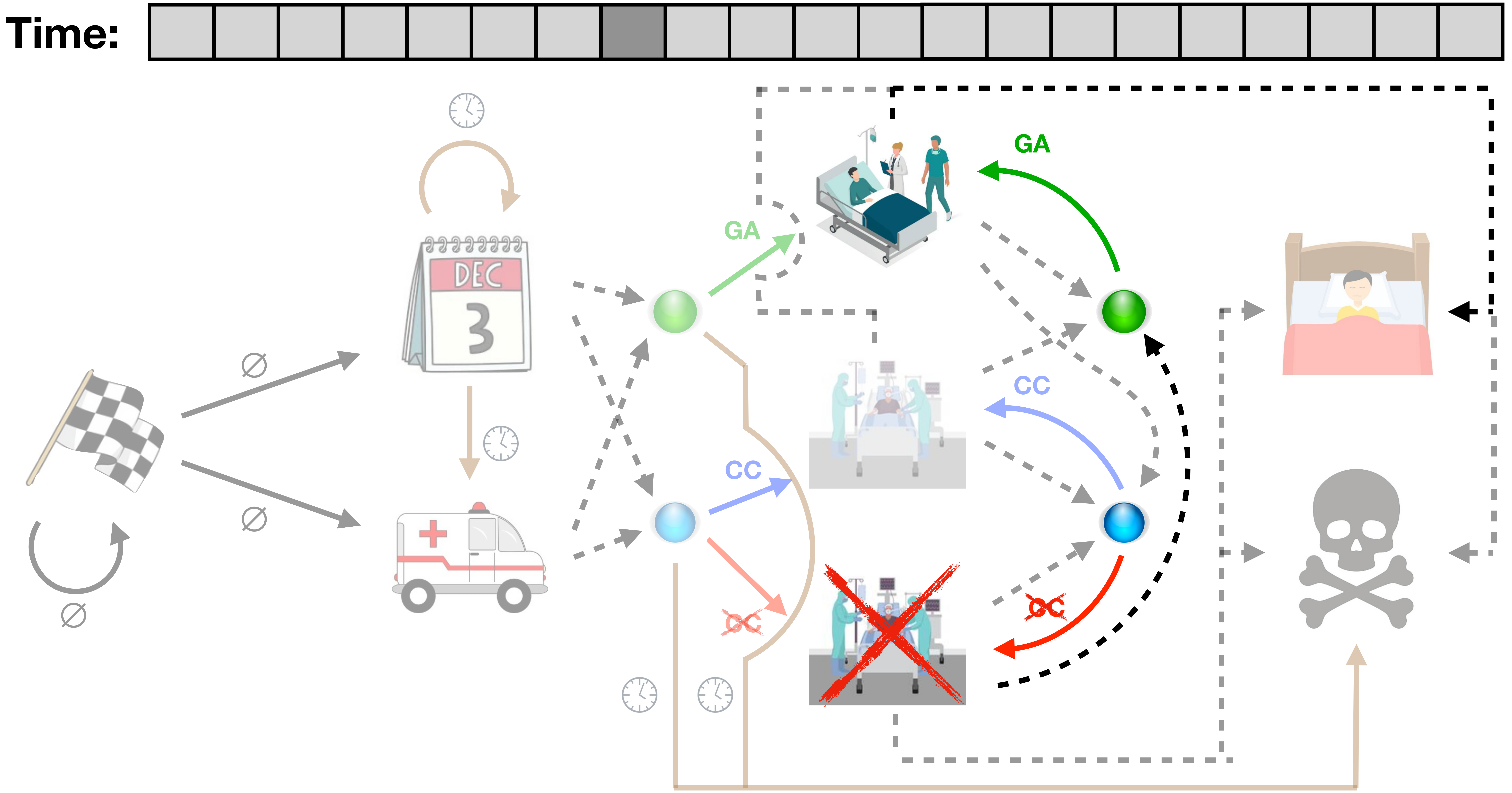

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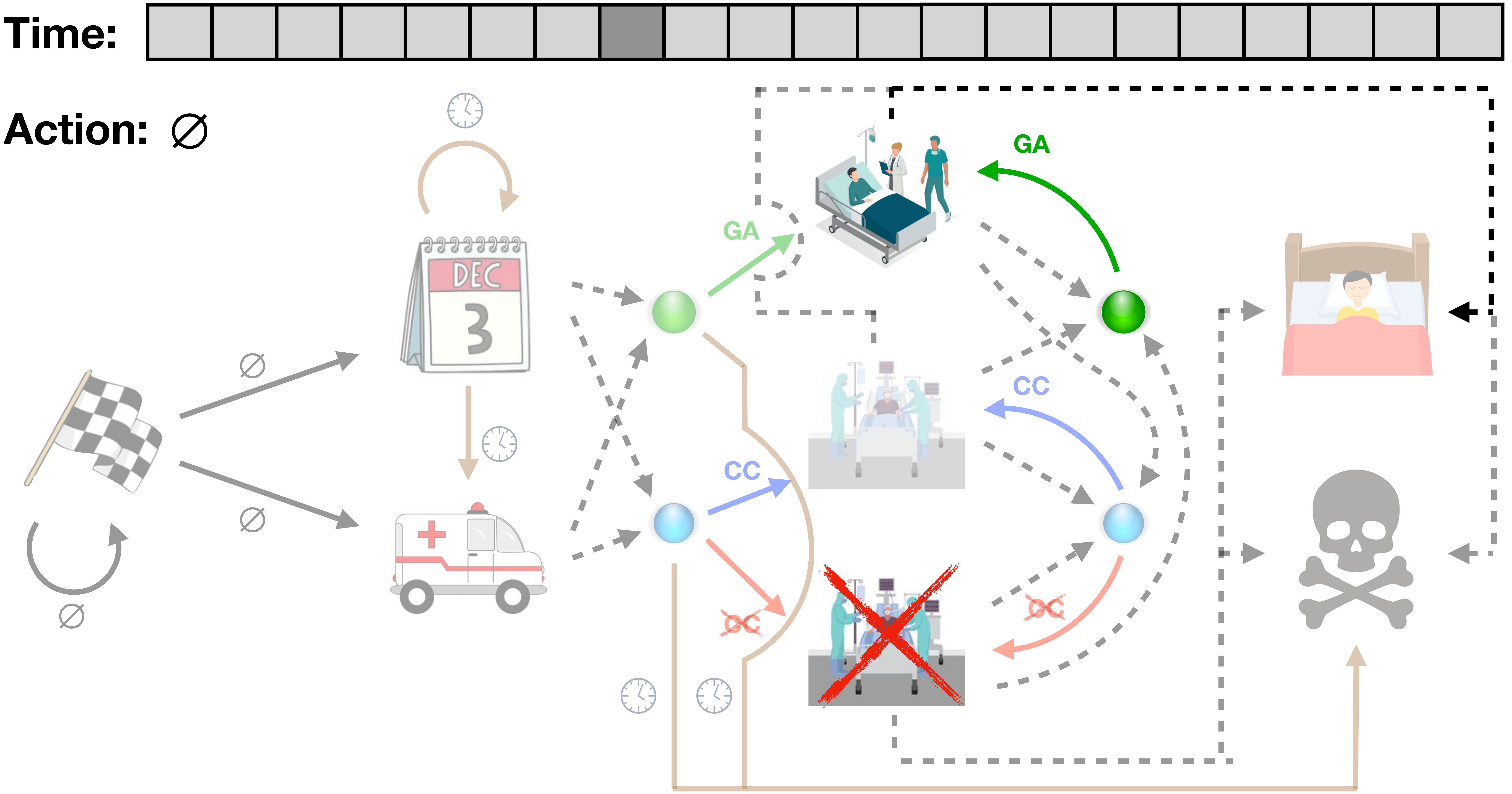

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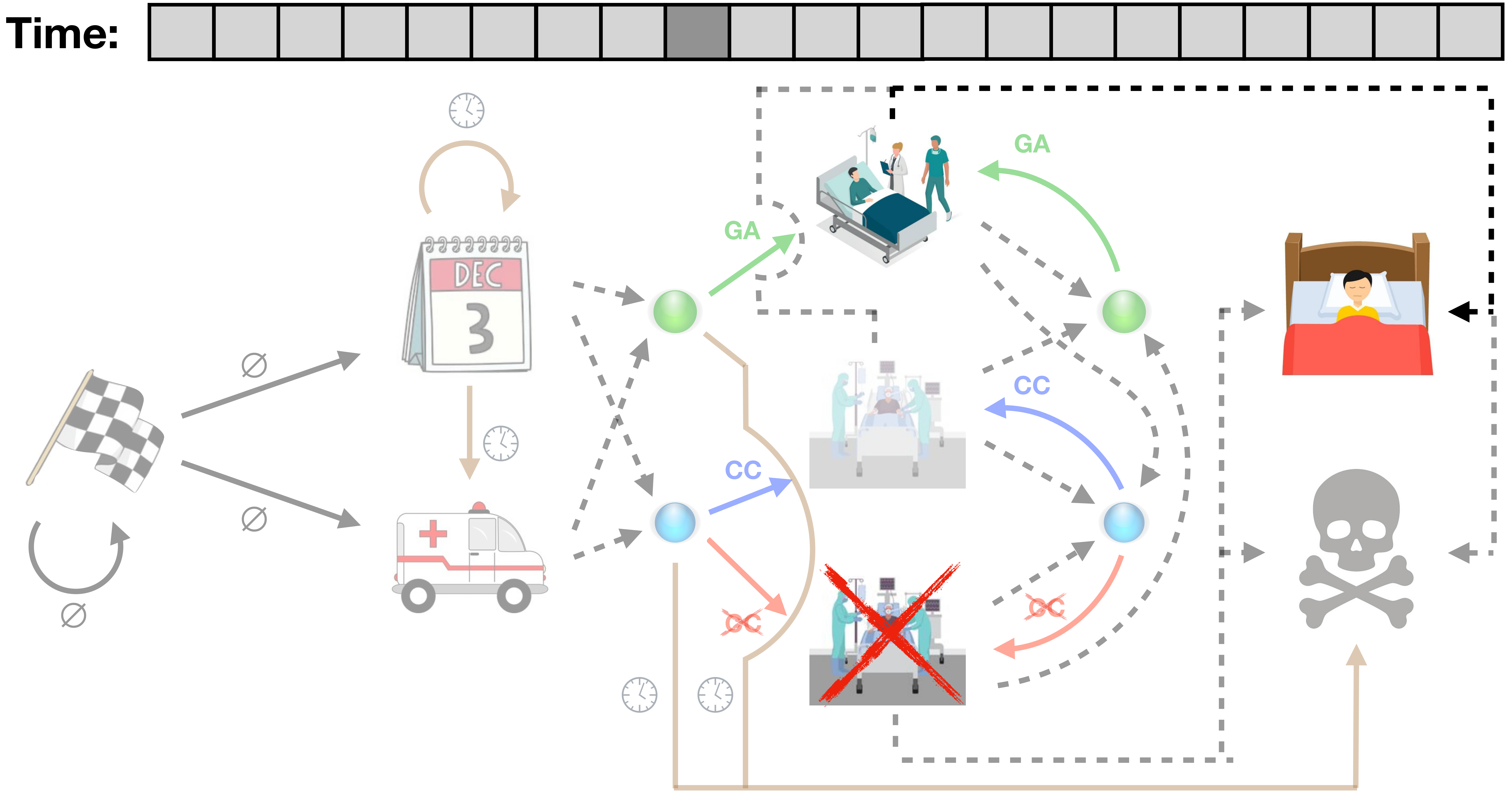

# Dynamic Programs (DPs): Extensive Model

**Can we combine all patient DPs to one large DP of the overall health system?**


**The state and action spaces satisfy**

$$|\mathcal{S}| = \prod_{i \in \mathcal{I}} |\mathcal{S}_i| \quad \textbf{and} \quad |\mathcal{A}| = \prod_{i \in \mathcal{I}} |\mathcal{A}_i|$$

**<u>In our case:</u> 10m patients à 15 states, 6 actions,**

$$|\mathcal{S}| = 15^{10,000,000} \quad \textbf{and} \quad |\mathcal{A}| = 6^{10,000,000}$$

**Any policy needs to map**

$$|\mathcal{S}| = 15^{10,000,000} \quad \textbf{to} \quad |\mathcal{A}| = 6^{10,000,000}$$

**Multiple patients of the same group can be modelled by a counting DP:**

**Counting DP**

For $n$ *iid* DPs $(\mathscr{S}, \mathscr{A}, q, p, r)$ over same $\mathscr{T}$, a DP $(\mathfrak{S}, \mathfrak{A}, \mathfrak{q}, \mathfrak{p}, \mathfrak{r})$ with:

- states $\mathfrak{S} = \{\sigma : \mathscr{S} \to \mathbb{N}_0\}$, actions $\mathfrak{A} = \{\alpha : \mathscr{S} \times \mathscr{A} \to \mathbb{N}_0\}$
- initial prob's $\mathfrak{q}(\sigma) = \dfrac{n!}{\prod_{s \in \mathscr{S}} \sigma(s)!} \cdot \prod_{s \in \mathscr{S}} q(s)^{\sigma(s)}$
- transition prob's

$$\mathfrak{p}_t(\sigma' \mid \sigma, \alpha) = \sum_{\theta \in \Gamma(\sigma, \alpha, \sigma')} \prod_{s \in \mathscr{S}} \prod_{a \in \mathscr{A}} \left[ \frac{\alpha(s,a)!}{\prod_{s' \in \mathscr{S}} \theta(s,a,s')!} \cdot \prod_{s' \in \mathscr{S}} p_t(s' \mid s, a)^{\theta(s,a,s')} \right]$$

- rewards $\mathfrak{r}_t(\sigma, \alpha) = \sum_{s \in \mathscr{S}} \sum_{a \in \mathscr{A}} r_t(s,a) \cdot \alpha(s,a)$

**Multiple patients of the same group can be modelled by a counting DP:**

**Counting DP**

For $n$ *iid* DPs $(\mathcal{S}, \mathcal{A}, q, p, r)$ over same $\mathcal{T}$, a D

**multinomial** $(n; q)$

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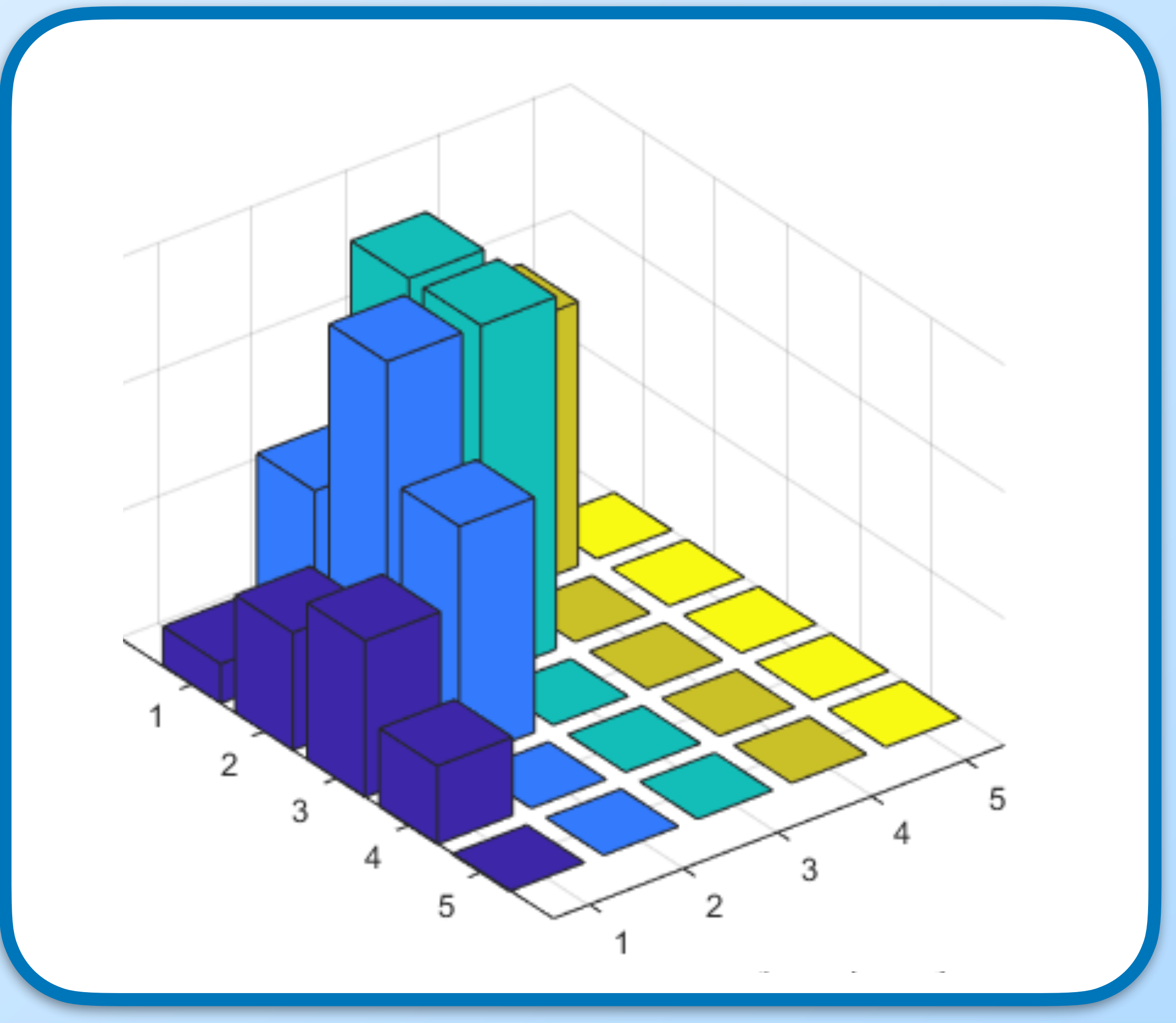

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- initial prob's $\mathfrak{q}(\sigma) = \frac{n!}{\prod_{s\in\mathcal{S}} \sigma(s)!} \cdot \prod_{s\in\mathcal{S}}$

**multinomial** $(\alpha(s,a); p_t(\,\cdot \mid s,a))$

- transition prob's

$$\mathfrak{p}_t(\sigma' \mid \sigma, \alpha) = \sum_{\theta\in\Gamma(\sigma,\alpha,\sigma')} \prod_{s\in\mathcal{S}} \prod_{a\in\mathcal{A}} \left[ \frac{\alpha(s,a)!}{\prod_{s'\in\mathcal{S}} \theta(s,a,s')!} \cdot \prod_{s'\in\mathcal{S}} p_t(s' \mid s,a)^{\theta(s,a,s')} \right]$$

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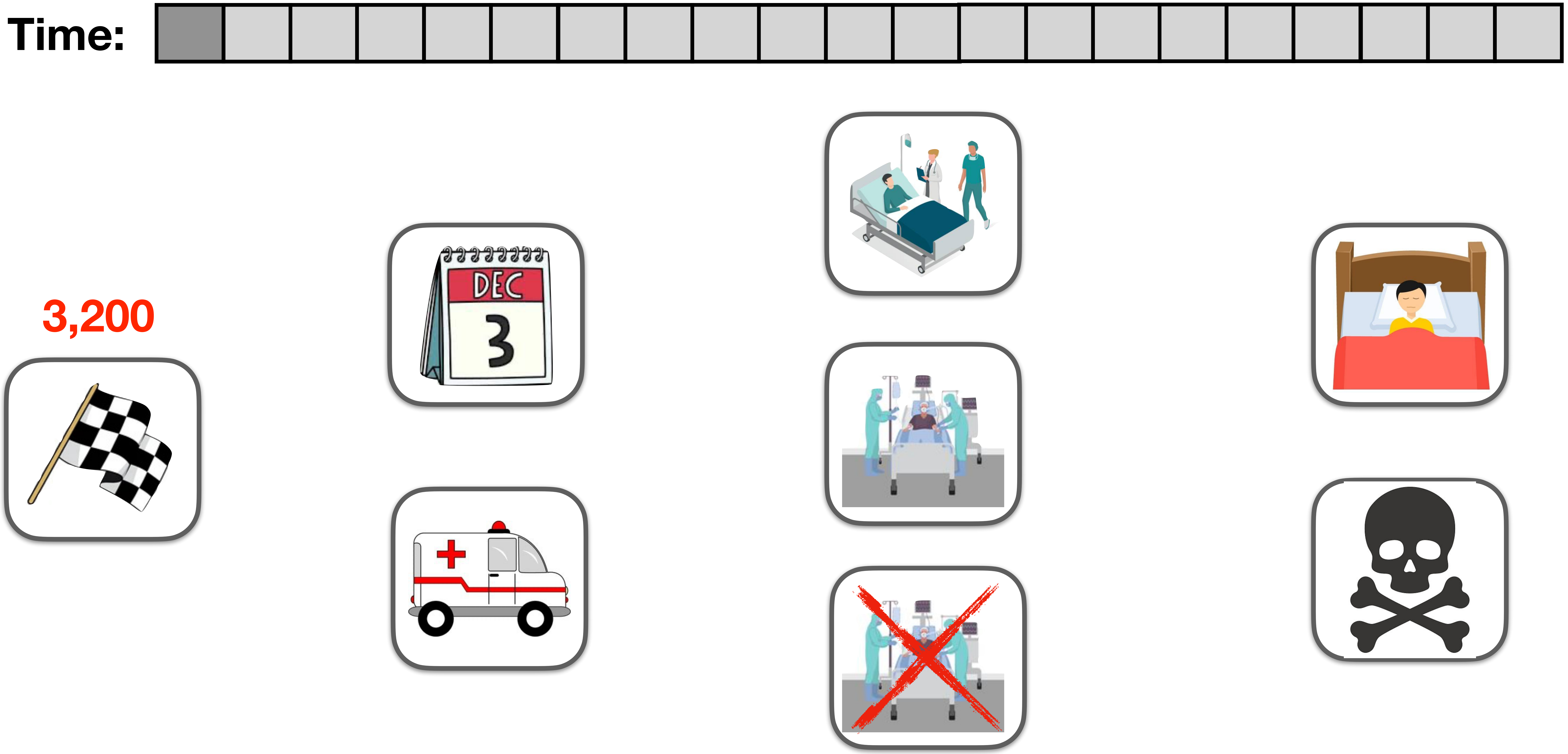
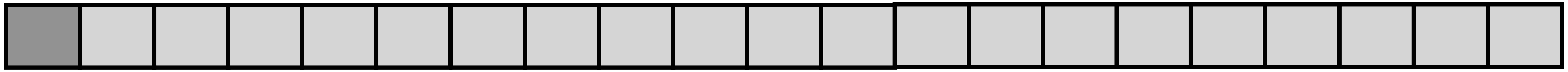
# Counting DPs



**Time:** 

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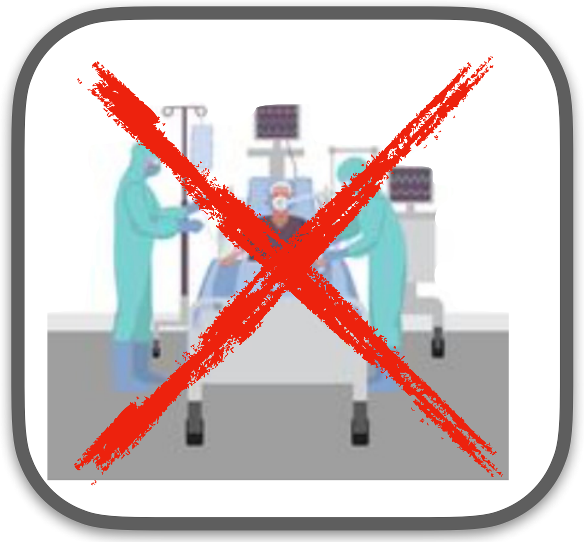

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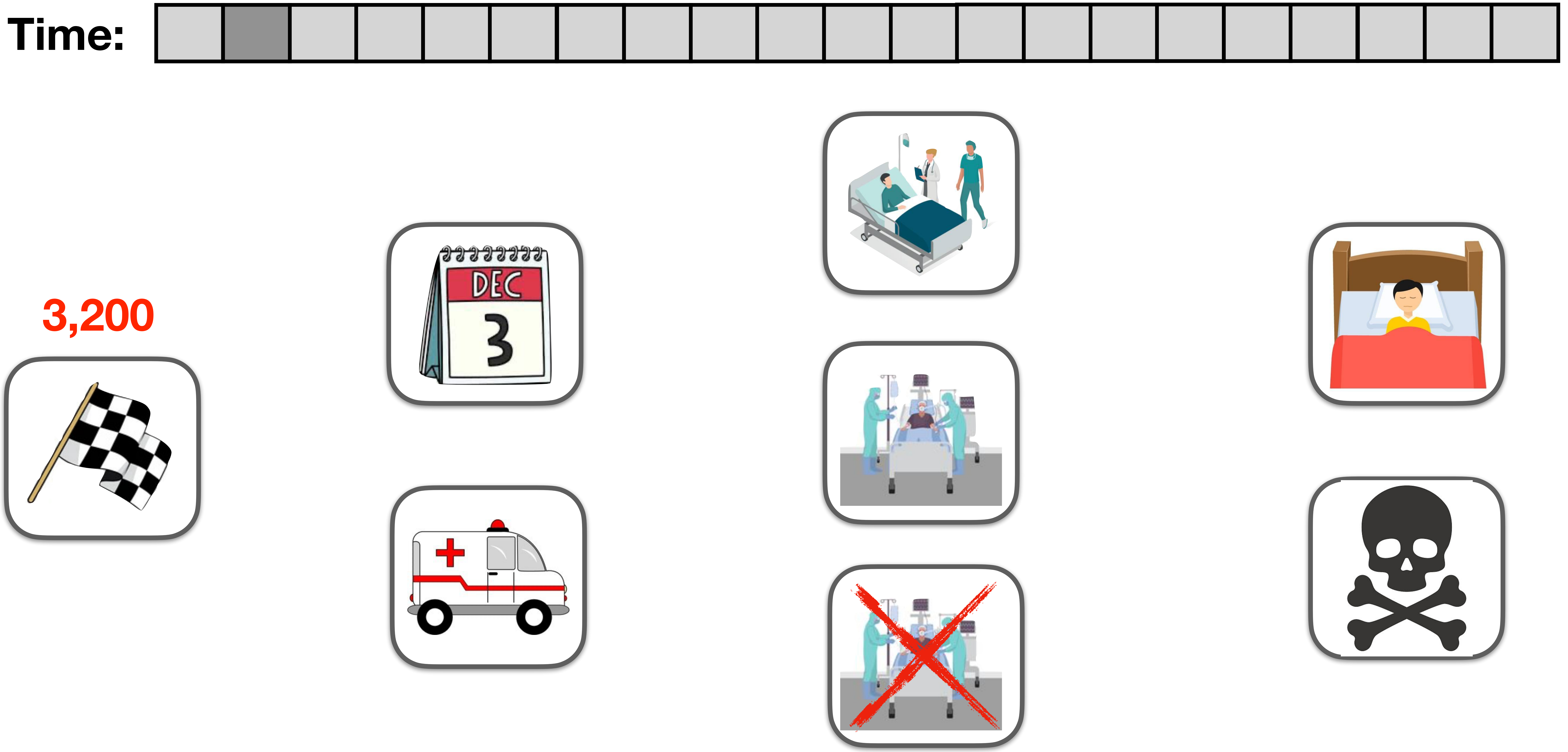

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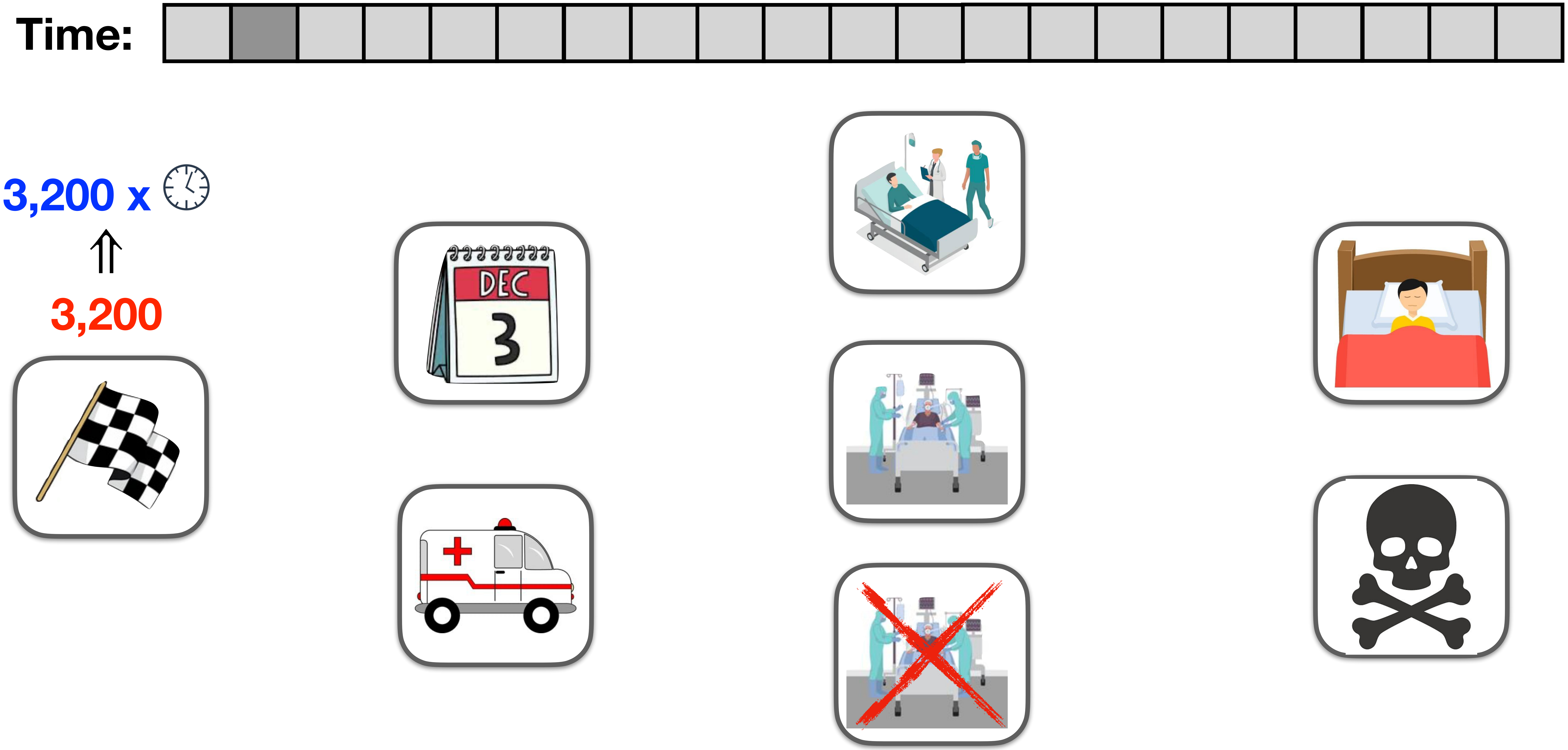

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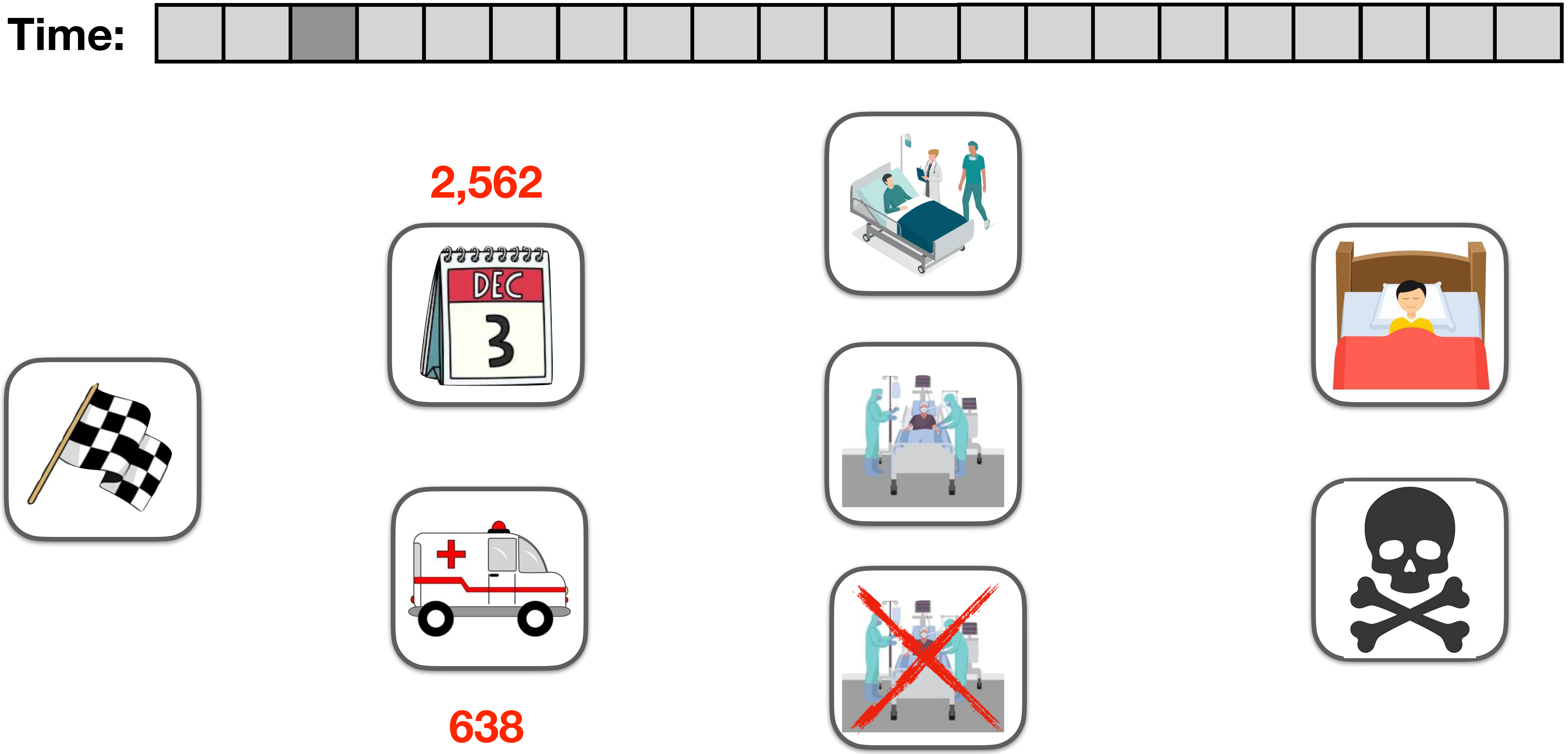

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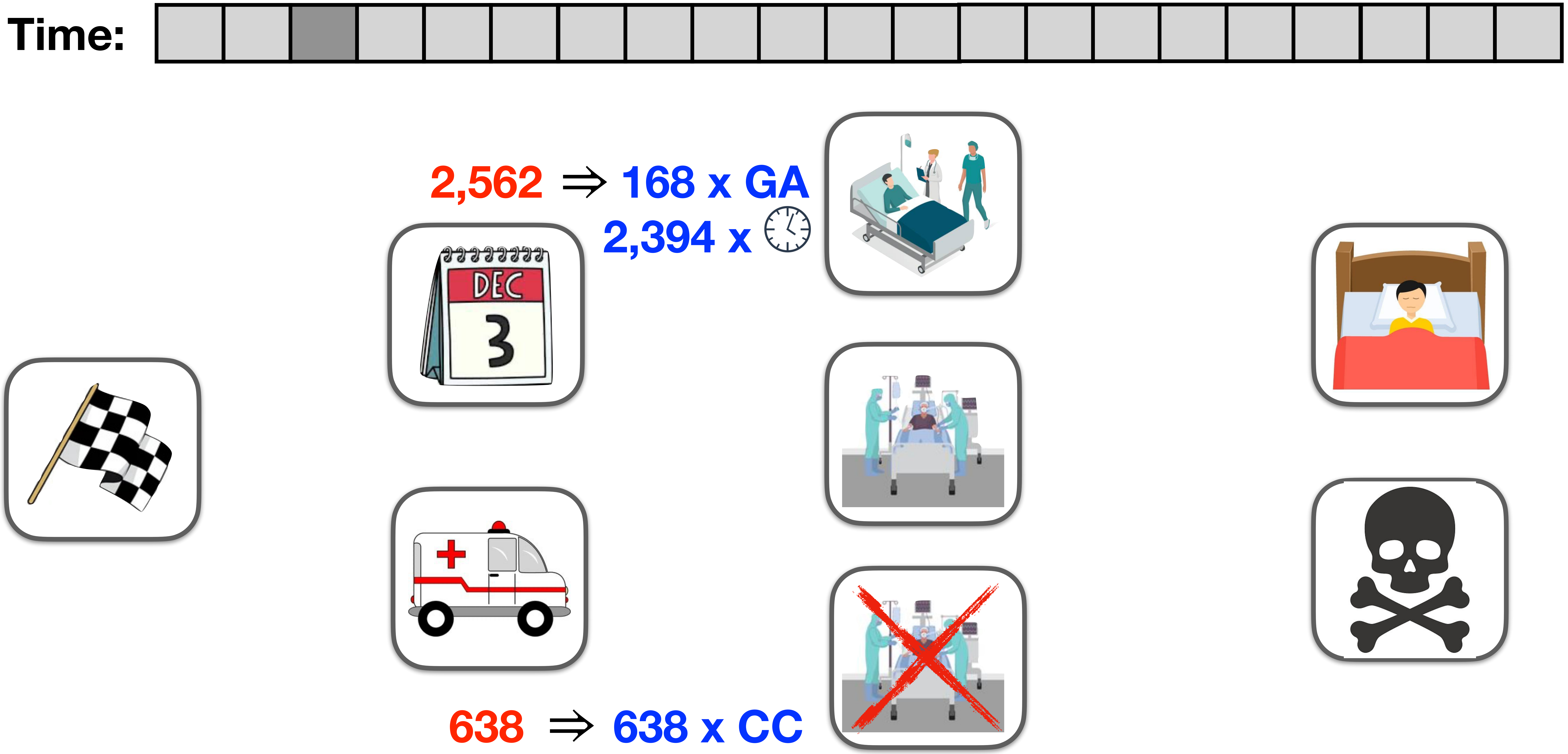

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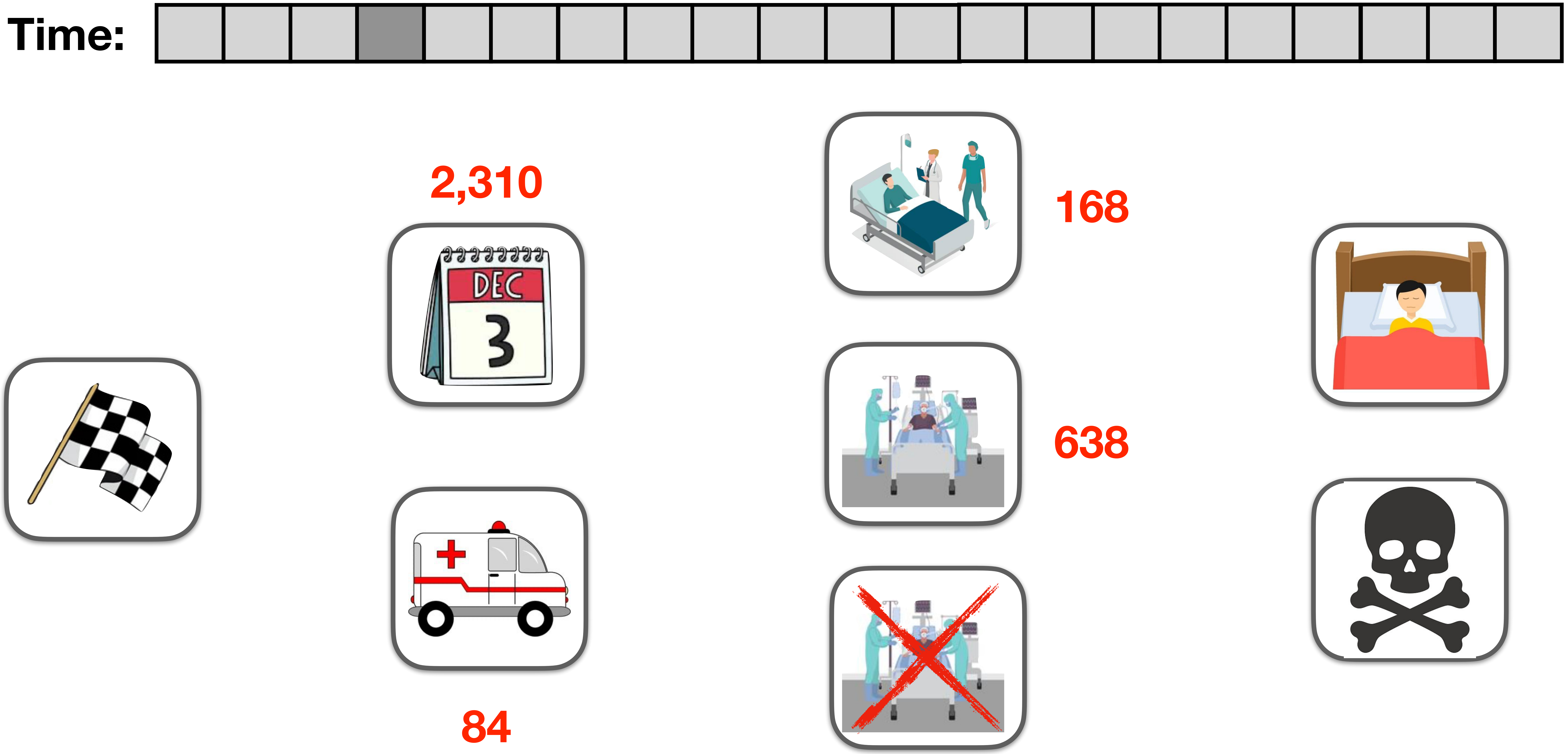

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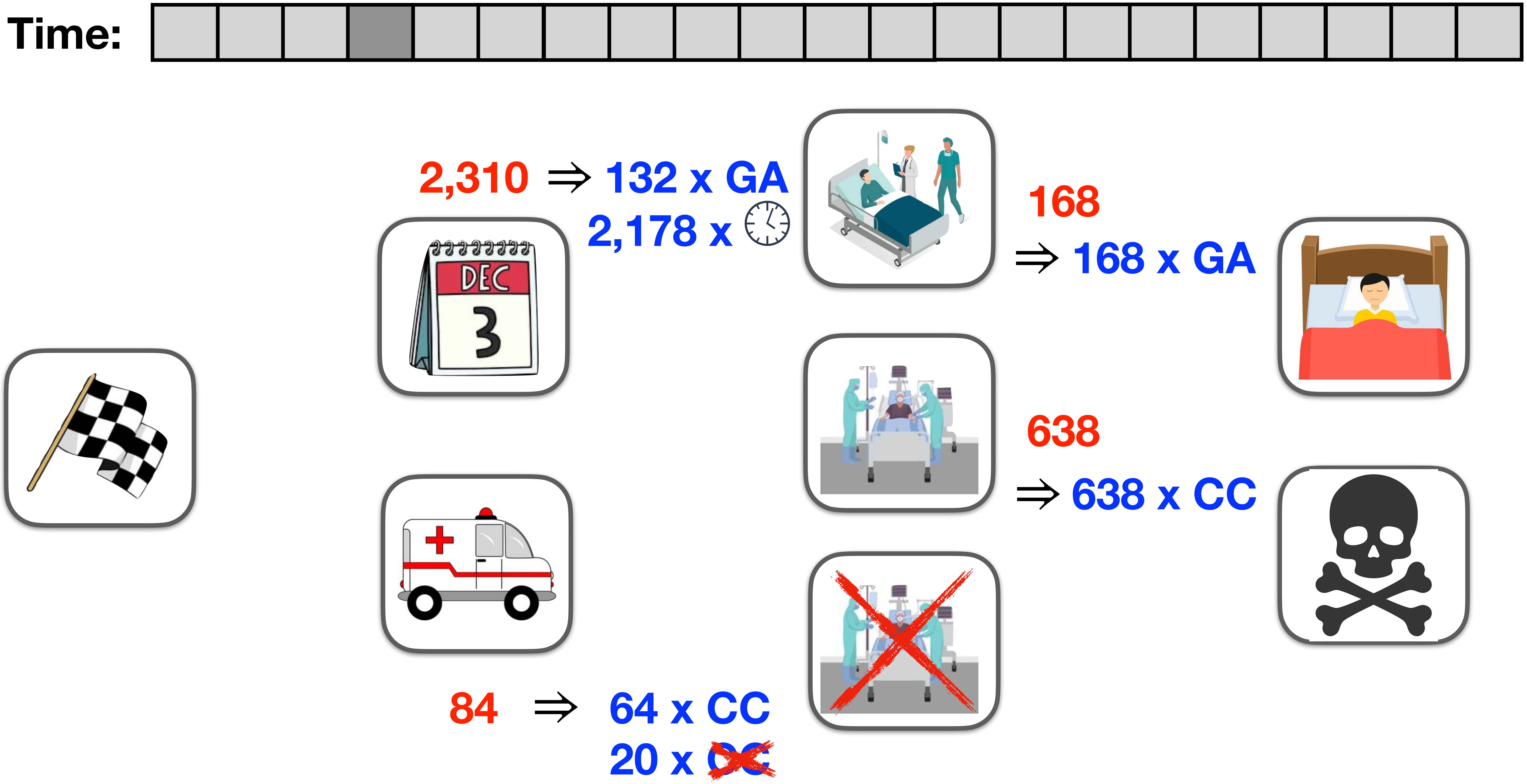

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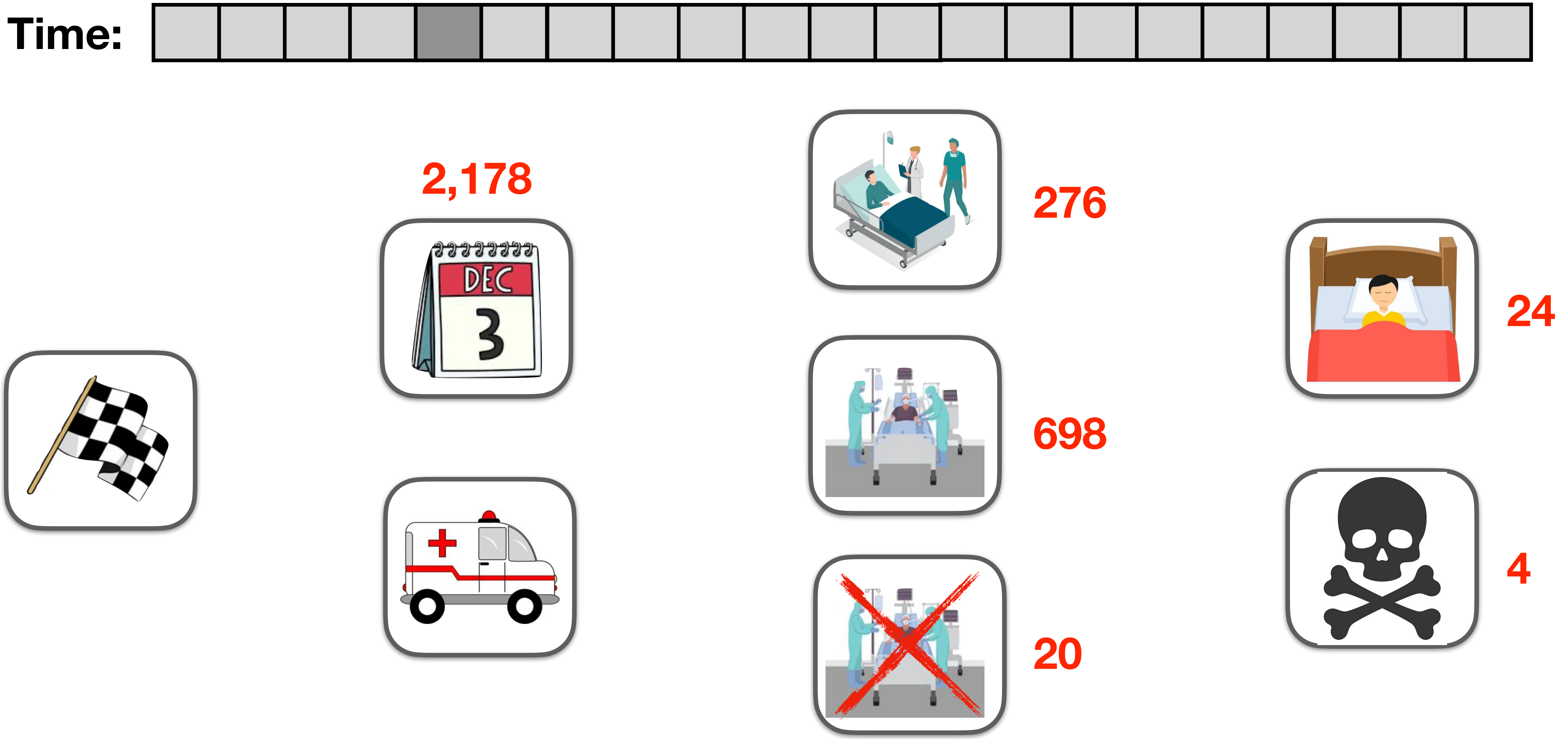


**Counting DPs achieve exponential compression over individual DPs:**

For $n$ patient DPs with $S$ states each, the state spaces scale according to

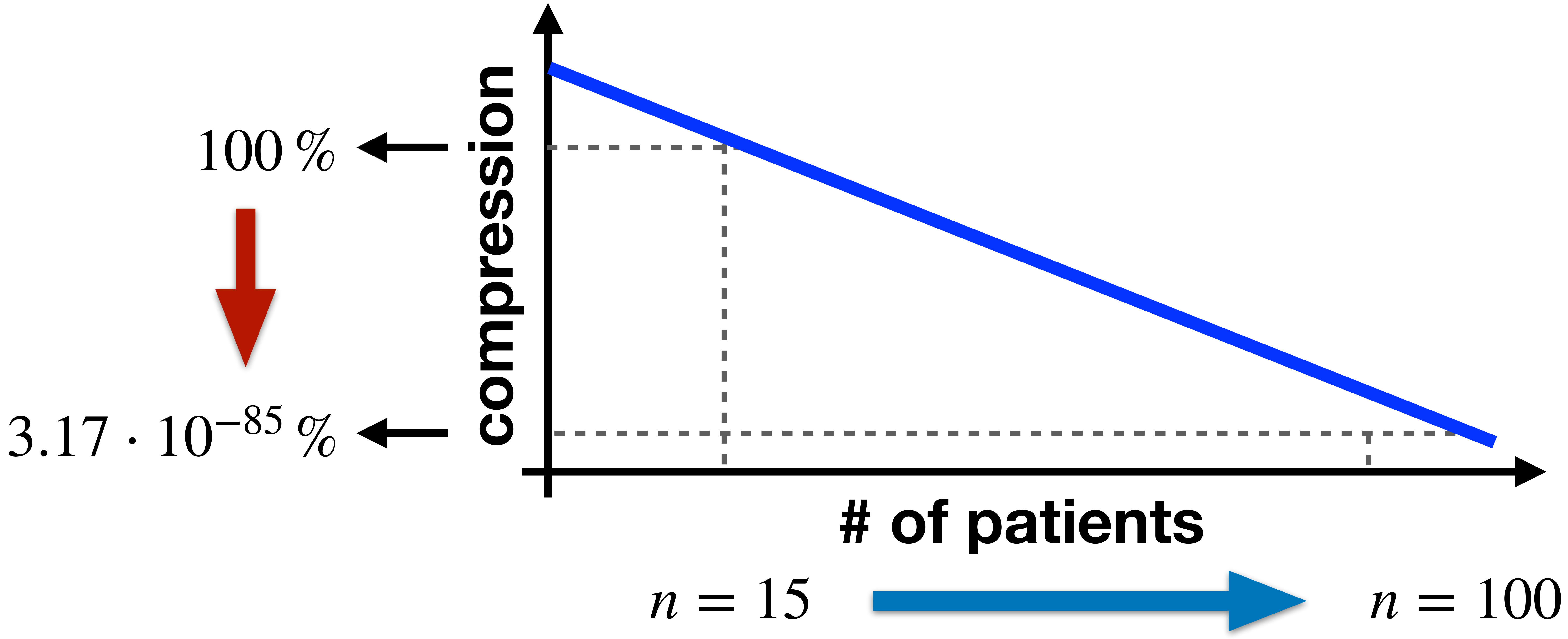
**Extensive Model**

$$|\mathcal{S}| = S^n$$

$$\Longrightarrow$$

**Counting DP**

$$|\mathfrak{S}| \approx n^S$$



**Can we combine all patient DPs to one large counting DP?**


**The state and action spaces satisfy**

$|\mathfrak{S}| \approx |\mathcal{I}|^{|\mathcal{S}_i|}$ **and** $|\mathfrak{A}| \approx |\mathcal{I}|^{|\mathcal{S}_i| \cdot |\mathcal{A}_i|}$

**<u>In our case:</u> 10m patients à 15 states, 6 actions,**

$|\mathfrak{S}| \approx 10{,}000{,}000^{15}$ **and** $|\mathfrak{A}| \approx 10{,}000{,}000^{15 \cdot 6}$

**Any policy needs to map**

$|\mathfrak{S}| \approx 10{,}000{,}000^{15}$ **to** $|\mathfrak{A}| \approx 10{,}000{,}000^{15 \cdot 6}$

**Multiple patients groups can be modelled by a weakly coupled counting DP:**

**Weakly Coupled Counting DP**

For the $J$ counting DPs $(\mathfrak{S}_j, \mathfrak{A}_j, \mathfrak{q}_j, \mathfrak{p}_j, \mathfrak{r}_j)$ with $n_j$ *iid* DPs each, over same $\mathcal{T}$, a DP $(\mathfrak{S}, \mathfrak{A}, \mathfrak{q}, \mathfrak{p}, \mathfrak{r})$ with:

- states $\mathfrak{S} = \times_j \mathfrak{S}_j$, actions $\mathfrak{A} = \times_j \mathfrak{A}_j$
- resource constraints $\sum_{j \in \mathcal{J}} \sum_{s \in \mathcal{S}_j} \sum_{a \in \mathcal{A}_j} c_{tlj}(s, a) \cdot \alpha_j(s, a) \leq b_{tl} \quad \forall l \in \mathcal{L}, \forall t \in \mathcal{T}$
- initial prob's $\mathfrak{q}(\sigma) = \prod_j \mathfrak{q}_j(\sigma_j)$, transitions $\mathfrak{p}_t(\sigma' \,|\, \sigma, \alpha) = \prod_j \mathfrak{p}_{jt}(\sigma_j' \,|\, \sigma_j, \alpha_j)$
- rewards $\mathfrak{r}_t(\sigma, \alpha) = \sum_j \mathfrak{r}_{jt}(\sigma_j, \alpha_j)$