# Witsenhausen Model for Leader-Follower Problems in Energy Management

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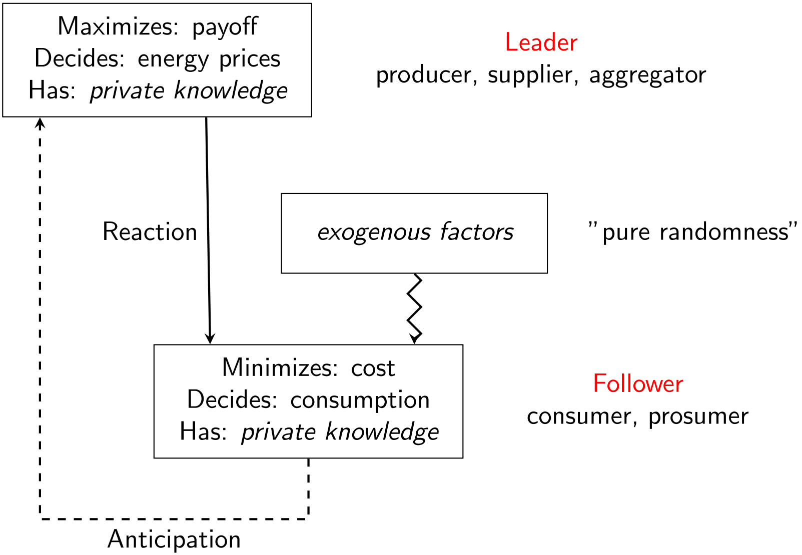
# What kind of problem are we looking at ?



# Why are we interested in this kind of problem?

- Before
  - Consumers were mostly passive users of energy
  - Energy was mainly generated from controllable sources (e.g. nuclear, gas)
  - Supply could be smoothly adjusted to match demand at any time
- Now
  - Consumers can now produce their own energy (e.g. solar panels)
  - Renewable energy sources depend on weather and cannot be easily controlled (e.g. wind, solar)
  - Communication technology make it possible to adjust demand in real time

## Demand response

Situations where customers change their consumption behaviors in response to price signals from the energy provider

# How to model this kind of problem?

- The information structure is sequential
  - Leader (e.g. electricity producer) plays first
  - Follower (e.g. consumer) reacts
- We focus on private knowledge
  - Leader's production cost
  - Follower's unwillingness to shift consumption
- We need to take "pure randomness" into account
  - Renewable energy production, demand, market prices
- We apply a versatile mathematical framework to handle problems with complex information structures
  - A W-model for decisions, uncertainty and information
  - A W-game for objective functions, beliefs and notions of equilibrium

# Outline of the presentation

Example: a time-of-use pricing problem

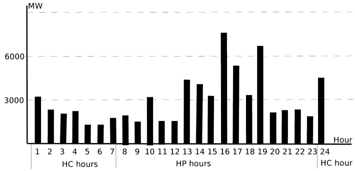
Nash-Stackelberg equilibrium in leader-follower W-games

Conclusion

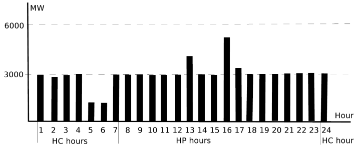
# Example of demand response program

## Time-of-use pricing

Price-based demand response program encouraging load-shifting: higher prices during peak hours and lower prices during off-peak hours




(a) Normal consumption




(b) Shifted consumption

Figure: Illustration of load-shifting [Alekseeva et al., 2019]

# Outline of the presentation

# Identification of the agents

An agent is a decision-maker taking only one action

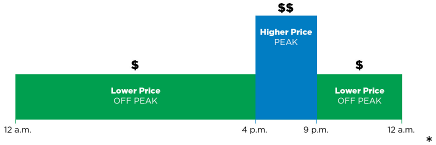
- 2 decisions: static setup
  - Deciding the electricity prices
  - Deciding how to shift the consumption

  $\Rightarrow$ 2 agents
  - Leader (agent): electricity producer
  - Follower (agent): consumer
- Example of dynamic setup
  - Deciding the electricity prices every month
  - Deciding how to shift the consumption every day

  $\Rightarrow$ More agents

# Schematic of time-of-use pricing

Decides: (peak, off-peak) prices




Leader (agent)
electricity producer

Decides: consumption shift

Follower (agent)
consumer

*https://www.cleanpowersf.org/tou

# Agents' actions and action sets

Each agent makes an action $u$ in a measurable space $(\mathcal{U}, \mathfrak{U})$
$\mathcal{U}$ is called the action set of an agent

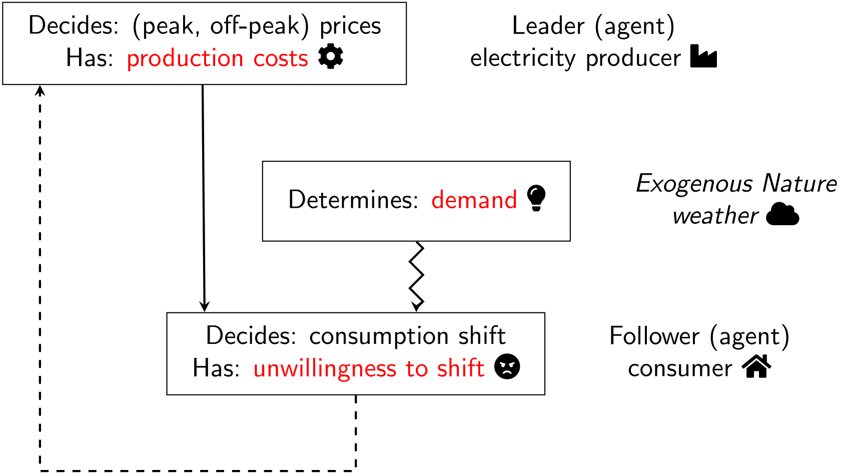
- Leader's action: (peak, off-peak) prices (€)

$$u^L = (\overline{u}^L, \underline{u}^L) \in \mathcal{U}^L = \{(x, y) \in \mathbb{R}^2 \mid x \geq y\} \subset \mathbb{R}^2$$

- Follower's action: consumption shift,
  i.e. fraction of consumption during (peak, off-peak) hours (%)

$$u^F = (\overline{u}^F, \underline{u}^F) \in \mathcal{U}^F = \{(\alpha, \beta) \in \mathbb{R}^2_+ \mid \alpha + \beta = 1\} \subset \mathbb{R}^2_+$$

# Players' and exogenous uncertainties



# Decomposition of Nature as a product

Nature contains everything that is not a decision

$$\Omega = \underbrace{\Omega^e}_{\substack{\text{exogenous}\\ \text{Nature}}} \times \underbrace{\Omega^L}_{\substack{\text{leader}\\ \text{type}}} \times \underbrace{\Omega^F}_{\substack{\text{follower}\\ \text{type}}}$$

- Exogenous Nature: electricity demand (kWh)

$$\omega^e \in \Omega^e = \mathbb{R}_+$$

- Leader type: unitary production cost (€/kWh)

$$\omega^L \in \Omega^L = \mathbb{R}_+$$

- Follower type: unwillingness to shift to off-peak hours (€/kWh)

$$\omega^F \in \Omega^F = \mathbb{R}_+$$

# Components of the upcoming objective functions

- Consumption (€)

$$\underbrace{\overline{u}^F \omega^e}_{\substack{\text{peak}\\ \text{demand}}} \cdot \underbrace{\overline{u}^L}_{\substack{\text{peak}\\ \text{price}}} + \underbrace{\underline{u}^F \omega^e}_{\substack{\text{off-peak}\\ \text{demand}}} \cdot \underbrace{\underline{u}^L}_{\substack{\text{off-peak}\\ \text{price}}}$$

- Production cost (€)

$$\underbrace{\omega^e}_{\substack{\text{total}\\ \text{demand}}} \cdot \underbrace{\omega^L}_{\substack{\text{unitary}\\ \text{production cost}}}$$

- Inconvenience cost (€)

$$\underbrace{\underline{u}^F \omega^e}_{\substack{\text{off-peak}\\ \text{demand}}} \cdot \underbrace{\omega^F}_{\substack{\text{unwillingness}\\ \text{to shift}}}$$

# Details of the configuration space

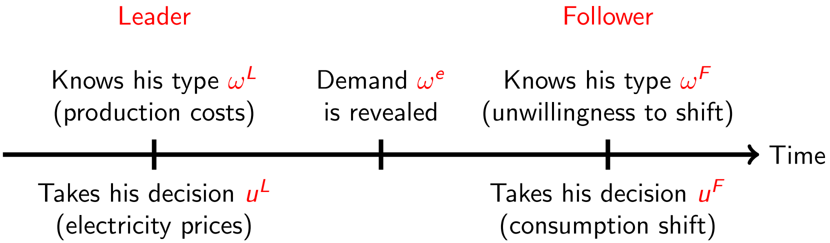
- Nature

$$\Omega = \underbrace{\mathbb{R}_+}_{\text{electricity demand}} \times \underbrace{\mathbb{R}_+}_{\text{unitary production cost}} \times \underbrace{\mathbb{R}_+}_{\text{unwillingness to shift}} = \mathbb{R}_+^3$$

Configuration space is the product space $\mathcal{H} = \Omega \times \mathcal{U}^L \times \mathcal{U}^F$

$$\mathcal{H} = \underbrace{\mathbb{R}_+^3}_{\text{Nature}} \times \underbrace{\{(x, y) \in \mathbb{R}^2 \mid x \geq y\}}_{\text{(peak, off-peak) prices}} \times \underbrace{\{(\alpha, \beta) \in \mathbb{R}_+^2 \mid \alpha + \beta = 1\}}_{\text{consumption shift}}$$

# Visualization of the information structure



# Leader's information field and strategies

The leader information field $\mathfrak{I}^L$ is a subfield of the $\sigma$-field associated with the configuration space $\mathfrak{H} = \mathfrak{G}^e \otimes \mathfrak{G}^L \otimes \mathfrak{G}^F \otimes \mathfrak{U}^L \otimes \mathfrak{U}^F$

$$\underbrace{\mathfrak{I}^L}_{\substack{\text{leader's}\\\text{information}\\\text{field}}} = \underbrace{\{\emptyset, \Omega^e\}}_{\substack{\text{cannot see}\\\text{consumer's}\\\text{demand}}} \otimes \underbrace{\mathfrak{G}^L}_{\substack{\text{knows his}\\\text{production}\\\text{cost}}} \otimes \underbrace{\{\emptyset, \Omega^F\}}_{\substack{\text{cannot see}\\\text{consumer's}\\\text{unwillingness}\\\text{to shift}}} \otimes \underbrace{\{\emptyset, \mathcal{U}^L\}}_{\substack{\text{absence of}\\\text{self-information}}} \otimes \underbrace{\{\emptyset, \mathcal{U}^F\}}_{\substack{\text{cannot see}\\\text{consumer's}\\\text{action}}}$$

A leader's strategy is a mapping $\lambda^L : (\mathcal{H}, \mathfrak{H}) \to (\mathcal{U}^L, \mathfrak{U}^L)$ measurable with respect to his information field $\mathfrak{I}^L$: $\lambda^{-1}(\mathfrak{U}^L) \subset \mathfrak{I}^L$

$$\underbrace{u^L}_{\substack{\text{electricity}\\\text{prices}}} = \underbrace{\lambda^L}_{\substack{\text{leader's}\\\text{strategy}}} \Big( \cancel{\omega^e}, \underbrace{\omega^L}_{\substack{\text{production}\\\text{costs}}}, \cancel{\omega^F}, \cancel{u^L}, \cancel{u^F} \Big)$$

# Follower's information field and strategies

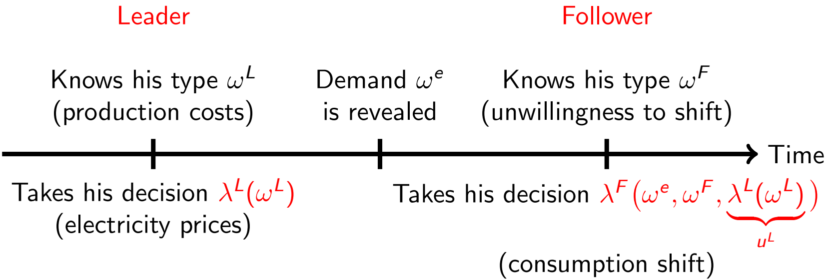
The follower information field $\mathfrak{I}^F$ is a subfield of the $\sigma$-field associated with the configuration space $\mathfrak{H} = \mathfrak{G}^e \otimes \mathfrak{G}^L \otimes \mathfrak{G}^F \otimes \mathfrak{U}^L \otimes \mathfrak{U}^F$

$$\underbrace{\mathfrak{I}^F}_{\text{follower's information field}} = \underbrace{\mathfrak{G}^e}_{\text{sees his demand}} \otimes \underbrace{\{\emptyset, \Omega^L\}}_{\text{cannot see producer's cost}} \otimes \underbrace{\mathfrak{G}^F}_{\text{knows his own unwillingness to shift}} \otimes \underbrace{\mathfrak{U}^L}_{\text{sees the electricity prices}} \otimes \underbrace{\{\emptyset, \mathcal{U}^F\}}_{\text{absence of self-information}}$$

A follower's strategy is a mapping $\lambda^F : (\mathcal{H}, \mathfrak{H}) \to (\mathcal{U}^F, \mathfrak{U}^F)$ measurable with respect to his information field $\mathfrak{I}^F$: $\lambda^{-1}(\mathfrak{U}^F) \subset \mathfrak{I}^F$

$$\underbrace{u^F}_{\text{consumption shift}} = \underbrace{\lambda^F}_{\text{follower's strategy}} \Big( \underbrace{\omega^e}_{\text{demand}}, \not{\omega}^L, \underbrace{\omega^F}_{\text{unwillingness to shift}}, \underbrace{u^L}_{\text{electricity prices}}, \not{u}^F \Big)$$

# A sequential (hence playable) information structure




When playability holds true, the solution map
is the mapping $S_{\lambda^L,\lambda^F} : \Omega \to \mathcal{H}$ which gives for every state of Nature
the unique outcome

$$S_{\lambda^L,\lambda^F}(\omega^e, \omega^L, \omega^F) = \Big(\omega^e, \omega^L, \omega^F, \underbrace{\lambda^L(\omega^L)}_{u^L}, \underbrace{\lambda^F\big(\omega^e, \omega^F, \lambda^L(\omega^L)\big)}_{u^F}\Big)$$

# What land have we covered? What comes next?

- Our problem is translated into a W-model
  - Agents: producer, consumer
  - Nature: production costs, unwillingness, demand
  - Sequential information structure: the consumer reacts to the producer
  - Strategies and solution map
- We now complete the W-model to have a W-game
  - Players
  - Objective functions
  - Beliefs

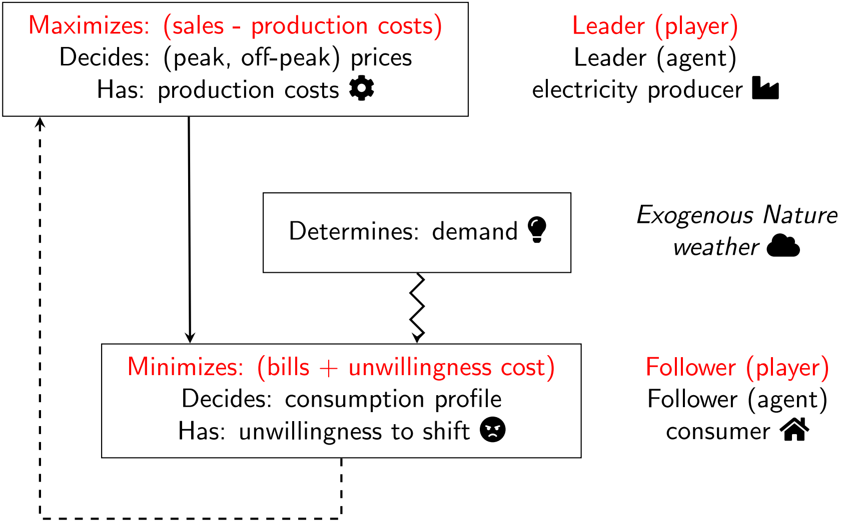
# Outline of the presentation

# Identification of the players

A player is an individual or a corporation, possibly taking several decisions, endowed with an objective function and a belief
To each player is associated a group of agents

- 2 players
  - Leader (player): electricity producer
  - Follower (player): consumer
- To each player corresponds a single agent
  $\Rightarrow$ same notations for the agent or the player
- Example of multi-leader-multi-follower setup
  - Several electricity producers
  - Several consumers

# Players' objective functions



# Expression of the objective functions

An objective function is a measurable function $j : \mathcal{H} \to \overline{\mathbb{R}} = \mathbb{R} \cup \{\pm\infty\}$ representing the player's preferences over the different outcomes

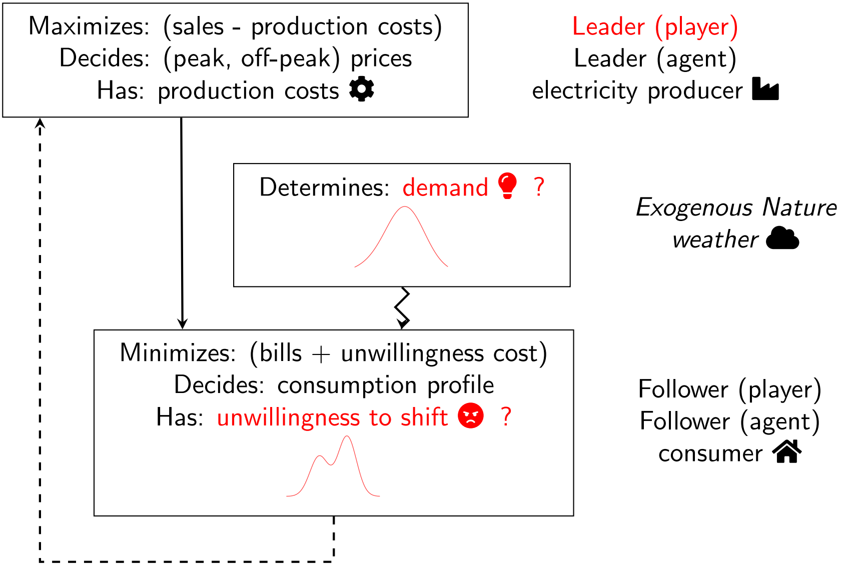
- Leader's payoff (maximization)

$$j^L(\omega^e, \omega^L, \cancel{\omega^F} u^L, u^F) = \underbrace{\overbrace{\overline{u}^F \omega^e}^{\text{peak demand}} \overbrace{\overline{u}^L}^{\text{peak price}} + \overbrace{\underline{u}^F \omega^e}^{\text{off-peak demand}} \overbrace{\underline{u}^L}^{\text{off-peak price}}}_{\text{sales}} - \underbrace{\overbrace{\omega^e}^{\text{total demand}} \overbrace{\omega^L}^{\text{unitary cost}}}_{\text{production cost}}$$

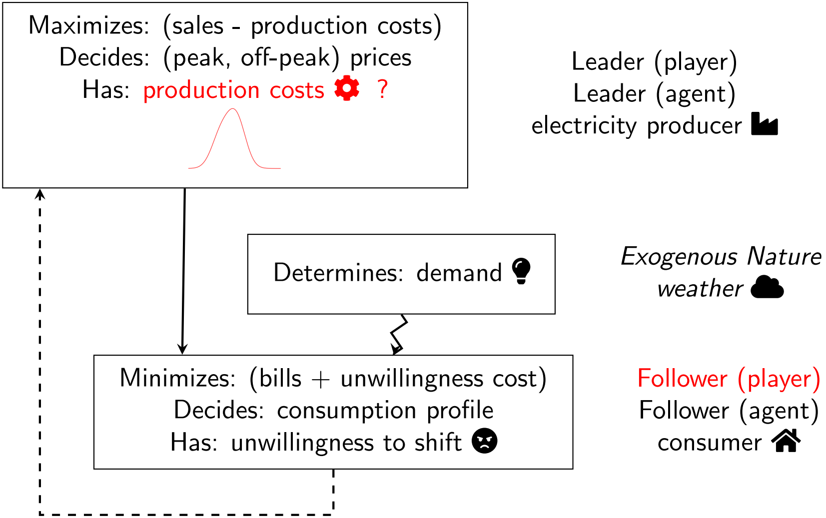
- Follower's cost (minimization)

$$j^F(\omega^e, \cancel{\omega^L}, \omega^F u^L, u^F) = \underbrace{\overbrace{\overline{u}^F \omega^e}^{\text{peak demand}} \overbrace{\overline{u}^L}^{\text{peak price}} + \overbrace{\underline{u}^F \omega^e}^{\text{off-peak demand}} \overbrace{\underline{u}^L}^{\text{off-peak price}}}_{\text{bills}} + \underbrace{\overbrace{\underline{u}^F \omega^e}^{\text{off-peak demand}} \overbrace{\omega^F}^{\text{unwillingness to shift}}}_{\text{inconvenience cost}}$$

# Leader's belief in time-of-use pricing



# Follower's belief in time-of-use pricing



# Decomposition of beliefs

Each player has a certain perception regarding uncertainty modelled by his belief, that is a probability distribution on

$$\Omega = \Omega^e \times \Omega^L \times \Omega^F$$

- Leader's belief

$$\beta^L = \underbrace{\beta_e^L}_{\text{distribution on consumer's demand}} \otimes \underbrace{\delta_{\{\omega^L\}}}_{\text{own type known}} \otimes \underbrace{\beta_F^L}_{\text{distribution on consumer's unwillingness to shift}}$$

- Follower's belief

$$\beta^F = \underbrace{\delta_{\{\omega^e\}}}_{\text{demand known}} \otimes \underbrace{\beta_L^F}_{\text{distribution on producer's cost}} \otimes \underbrace{\delta_{\{\omega^F\}}}_{\text{own type known}}$$

# Normal form W-games

- Strategies are the heart of normal form games
  - $\Lambda^L$: set of leader's strategies
  - $\Lambda^F$: set of follower's strategies

The normal form objective function is a function $J : \Lambda^L \times \Lambda^F \to \overline{\mathbb{R}}$ giving what a player can expect to gain (or lose) from a strategy profile

| $L$ , $F$ | $\ldots$ | $\lambda^F$ | $\ldots$ |
|---|---|---|---|
| $\ldots$ | | | |
| $\lambda^L$ | | $J^L(\lambda^L, \lambda^F)$ , $J^F(\lambda^L, \lambda^F)$ | |
| $\ldots$ | | | |

Table: Normal form representation of a W-game

# Expression of normal form objective functions

When working with beliefs, the normal form objective function is the average gain (or loss) of a strategy profile

$$J(\lambda^L,\lambda^F) = \mathbb{E}_\beta\Big[ \underbrace{j \circ S_{\lambda^L,\lambda^F}}_{\Omega \xrightarrow{S_{\lambda^L,\lambda^F}} \mathcal{H} \xrightarrow{j} \overline{\mathbb{R}}} \Big] = \int_\Omega \big(j \circ S_{\lambda^L,\lambda^F}\big)(\omega)\, \mathrm{d}\beta(\omega)$$

- Leader's normal form payoff (maximization)

$$J^L(\lambda^L,\lambda^F) = \int_\Omega j^L\Big(\underbrace{\omega^e,\omega^L,\cancel{\omega^F},\lambda^L(\omega^L),\lambda^F\big(\omega^e,\omega^F,\lambda^L(\omega^L)\big)}_{S_{\lambda^L,\lambda^F}}\Big)\,\mathrm{d}\beta^L(\omega)$$

- Follower's normal form cost (minimization)

$$J^F(\lambda^L,\lambda^F) = \int_\Omega j^F\Big(\underbrace{\omega^e,\cancel{\omega^L},\omega^F,\lambda^L(\omega^L),\lambda^F\big(\omega^e,\omega^F,\lambda^L(\omega^L)\big)}_{S_{\lambda^L,\lambda^F}}\Big)\,\mathrm{d}\beta^F(\omega)$$

# Focus on asymmetric knowledge: game data

Player's data refers to the player's objective function and belief
W-game data is the collection of the players' data and write

$$d = \Big( \underbrace{(j^L, \beta^L)}_{d^L}, \underbrace{(j^F, \beta^F)}_{d^F} \Big)$$

- Leader's normal form payoff (maximization)

$$J^L(\lambda^L, \lambda^F; \underbrace{d^L}_{\text{data}}) = \int_\Omega \underbrace{j^L\Big(\omega^e, \omega^L, \lambda^L(\omega^L), \lambda^F\big(\omega^e, \omega^F, \lambda^L(\omega^L)\big)\Big)}_{\text{objective function}} \,\mathrm{d}\, \underbrace{\beta^L(\omega)}_{\text{belief}}$$

- Follower's normal form cost (minimization)

$$J^F(\lambda^F, \lambda^L; \underbrace{d^F}_{\text{data}}) = \int_\Omega \underbrace{j^F\Big(\omega^e, \omega^F, \lambda^L(\omega^L), \lambda^F\big(\omega^e, \omega^F, \lambda^L(\omega^L)\big)\Big)}_{\text{objective function}} \,\mathrm{d}\, \underbrace{\beta^F(\omega)}_{\text{belief}}$$

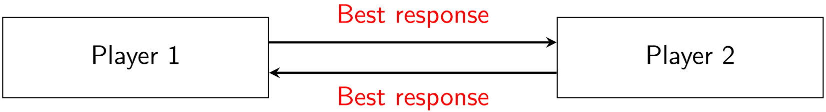
# What land have we covered? What comes next?

- Time-of-use pricing problem is now a W-game
  - Objective functions: producer's payoff, consumer's cost
  - Decomposition of beliefs
- The W-game can be written in normal form
  - Normal form objective function
  - Everything in the strategies
- We focus on W-game data to model asymmetric knowledge
- We now move to translating game theory equilibrium concepts in the language of W-games
  - Best response and Nash equilibrium
  - Stackelberg strategy and Nash-Stackelberg equilibrium

# Outline of the presentation

# Illustration of a Nash equilibrium

A player plays a best response if he chooses a strategy that maximizes (or minimizes) his own objective function, given the strategies selected by the others
A Nash equilibrium is when each player's strategy is a best response to the strategies of the other players




- Most common notion for "solving" a game
- Stable situation: no player has an incentive to deviate unilaterally
- Example: a group of producers can play a Nash equilibrium

# Nash equilibrium in leader-follower W-games

- Leader's best responses (maximization)

$$\Lambda_N^L(\lambda^F; d^L) = \arg\max_{\lambda^L \in \Lambda^L} J^L(\lambda^L, \lambda^F; d^L) \subset \Lambda^L$$

- Follower's best responses (minimization)

$$\Lambda_N^F(\lambda^L; d^F) = \arg\min_{\lambda^F \in \Lambda^F} J^F(\lambda^L, \lambda^F; d^F) \subset \Lambda^F$$

## Nash equilibrium

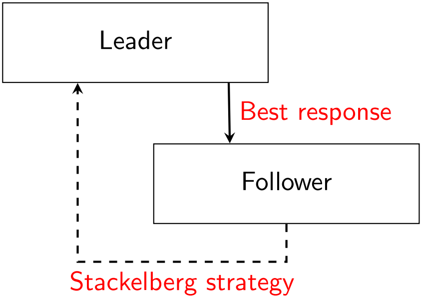
A strategy profile $(\lambda^L, \lambda^F) \in \Lambda^L \times \Lambda^F$ that satisfies

$$\begin{cases} \lambda^L \in \Lambda_N^L(\lambda^F; d^L) : \text{the leader plays a best response} \\ \lambda^F \in \Lambda_N^F(\lambda^L; d^F) : \text{the follower plays a best response} \end{cases}$$

# Illustration of a Nash-Stackelberg equilibrium

A player plays a Stackelberg strategy if he chooses a strategy that maximizes (or minimizes) his own objective function, assuming the others play a best response

A Nash-Stackelberg equilibrium is when one player plays a best response and the other anticipates by choosing a Stackelberg strategy



# Different types of Stackelberg strategies

- Stackelberg strategy is for the leader (maximization)
- Problem: multiplicity of best responses for the follower
- Optimistic Stackelberg strategies: the follower chooses the best response that is most advantageous for the leader

$$\Lambda_S^L(d^L, d^F) = \arg\max_{\lambda^L \in \Lambda^L} \sup_{\lambda^F \in \Lambda_N^F(\lambda^L; d^F)} J^L(\lambda^L, \lambda^F; d^L) \subset \Lambda^L$$

- Pessimistic Stackelberg strategies: the follower chooses the best response that is least advantageous for the leader

$$\Lambda_S^L(d^L, d^F) = \arg\max_{\lambda^L \in \Lambda^L} \inf_{\lambda^F \in \Lambda_N^F(\lambda^L; d^F)} J^L(\lambda^L, \lambda^F; d^L) \subset \Lambda^L$$

- Existence of intermediate formulations (between optimistic and pessimistic)

# Nash-Stackelberg equilibrium in leader-follower W-games

## Nash-Stackelberg equilibrium

A strategy profile $(\lambda^L, \lambda^F) \in \Lambda^L \times \Lambda^F$ that satisfies

$$\begin{cases} \lambda^L \in \Lambda^L_S(d^L, d^F) : \text{the leader plays a Stackelberg strategy} \\ \lambda^F \in \Lambda^F_N(\lambda^L; d^F) : \text{the follower plays a best response} \end{cases}$$

- Writes as a bilevel optimization problem (optimistic formulation)

$$\max_{\lambda^L \in \Lambda^L} \sup_{\lambda^F \in \Lambda^F} J^L(\lambda^L, \lambda^F; d^L) \qquad \text{(UL)}$$

$$\text{subject to} \quad \lambda^F \in \min_{\lambda^F \in \Lambda^F} J^F(\lambda^L, \lambda^F; d^F) \qquad \text{(LL)}$$

- Upper-Level problem (UL): leader's problem (maximization)
- Lower-Level problem (LL): follower's problem (minimization)
- Ambiguous knowledge of the W-game data

# What land have we covered? What comes next?

- We conducted the entire study on a simple example
- We revisited key concepts of game theory in W-games
  - Best response
  - Nash equilibrium
- We explored other concepts for leader-follower games
  - Stackelberg strategy
  - Nash-Stackelberg equilibrium: link with bilevel optimisation
- We raised the question of the W-game data

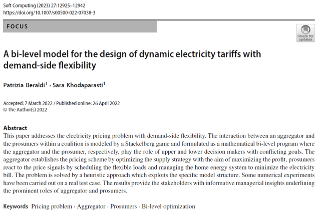
# Outline of the presentation

Example: a time-of-use pricing problem

Nash-Stackelberg equilibrium in leader-follower W-games

Conclusion

# W-games can easily deal with similar energy management problems



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FOCUS

**A bi-level model for the design of dynamic electricity tariffs with demand-side flexibility**

Patrizia Beraldi[1] · Sara Khodaparasti[1]

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**Abstract**
This paper addresses the electricity pricing problem with demand-side flexibility. The interaction between an aggregator and the prosumers within a coalition is modeled by a Stackelberg game and formulated as a mathematical bi-level program where the aggregator and the prosumer, respectively, play the role of upper and lower decision makers with conflicting goals. The aggregator establishes the pricing scheme by optimizing the supply strategy with the aim of maximizing the profit, prosumers react to the price signals by scheduling the flexible loads and managing the home energy system to minimize the electricity bill. The problem is solved by a heuristic approach which exploits the specific model structure. Some numerical experiments have been carried out on a real test case. The results provide the stakeholders with informative managerial insights underlining the prominent roles of aggregator and prosumers.

**Keywords** Pricing problem · Aggregator · Prosumers · Bi-level optimization

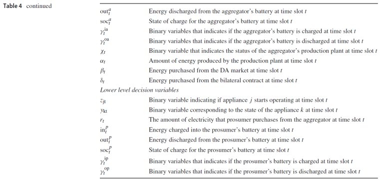

**Table 4** continued

| | |
|---|---|
| $\text{out}_t^a$ | Energy discharged from the aggregator's battery at time slot $t$ |
| $\text{soc}_t^a$ | State of charge for the aggregator's battery at time slot $t$ |
| $\gamma_t^{\text{ia}}$ | Binary variables that indicates if the aggregator's battery is charged at time slot $t$ |
| $\gamma_t^{\text{oa}}$ | Binary variables that indicates if the aggregator's battery is discharged at time slot $t$ |
| $\chi_t$ | Binary variable that indicates the status of the aggregator's production plant at time slot $t$ |
| $\alpha_t$ | Amount of energy produced by the production plant at time slot $t$ |
| $\beta_t$ | Energy purchased from the DA market at time slot $t$ |
| $\delta_t$ | Energy purchased from the bilateral contract at time slot $t$ |
| *Lower level decision variables* | |
| $z_{\text{jt}}$ | Binary variable indicating if appliance $j$ starts operating at time slot $t$ |
| $y_{\text{kt}}$ | Binary variable corresponding to the state of the appliance $k$ at time slot $t$ |
| $r_t$ | The amount of electricity that prosumer purchases from the aggregator at time slot $t$ |
| $\text{in}_t^p$ | Energy charged into the prosumer's battery at time slot $t$ |
| $\text{out}_t^p$ | Energy discharged from the prosumer's battery at time slot $t$ |
| $\text{soc}_t^p$ | State of charge for the prosumer's battery at time slot $t$ |
| $\gamma_t^{\text{ip}}$ | Binary variables that indicates if the prosumer's battery is charged at time slot $t$ |
| $\gamma_t^{\text{op}}$ | Binary variables that indicates if the prosumer's battery is discharged at time slot $t$ |

Figure: Extracts from [Beraldi and Khodaparasti, 2022]

- Leader: aggregator
- Richer action sets $\mathcal{U}^L = \{\text{PV panels production}, \text{plant production}, \text{purchase on the market}, \text{battery management}, \text{bilateral contracts}\}$
- And richer private knowledge $\Omega^L = \{\text{plant production bounds}, \text{battery state bounds}, \text{market prices}, \text{production costs}\}$
- Same for the follower who is a prosumer (profesional + consumer)

# W-games can easliy deal with dynamic problems

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A Bilevel Stochastic Programming Approach for Retailer Futures Market Trading

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***Abstract*—This paper presents a bilevel programming approach to solve the medium-term decision-making problem faced by a power retailer. A retailer decides its level of involvement in the futures market and in the pool as well as the selling price offered to its potential clients with the goal of maximizing the expected profit at a given risk level. Uncertainty on future pool prices, client demands, and rival-retailer prices is accounted for via stochastic programming. Unlike in previous approaches, client response to retail price and competition among rival retailers are both explicitly considered in the proposed bilevel model. The resulting nonlinear bilevel programming formulation is transformed into an equivalent single-level mixed-integer linear programming problem by replacing the lower-level optimization by its Karush–Kuhn–Tucker optimality conditions and converting a number of nonlinearities to linear equivalents using some well-known integer algebra results. A realistic case study is solved to illustrate the efficient performance of the proposed methodology.**

***Index Terms*—Bilevel programming, futures market, power retailer, risk, stochastic programming.**

$X^0_{e,s}$ Percentage of the demand of client group e initially supplied by retailer s.

$\alpha$ Confidence level used in the calculation of the CVaR.

$\beta$ Weighting factor.

$\gamma_e$ Parameter representing the relationship between the pool price and the demand of client group e.

$\lambda^F_{f,j}$ Price of block j of the forward contracting curve of contract f [€/MWh].

$\lambda^P_t(\omega)$ Pool price in period t and scenario $\omega$ [€/MWh].

$\hat{\lambda}^P_t$ Expected pool price in period t [€/MWh].

$\lambda^S_{e,s}(\xi)$ Selling price offered by retailer s to client group e in scenario $\xi$ [€/MWh].

$\pi(\omega)$ Probability of occurrence of pool price and client demand scenario $\omega$.

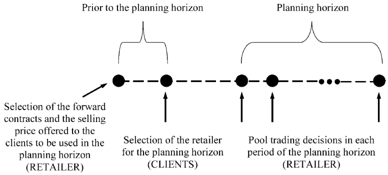


Fig. 1. Decision-making process.

Figure: Extracts from [Carrión, Arroyo, and Conejo, 2009]

- Agents are points in the arrow of time $\mathcal{U}^L = \prod_{t \in \mathcal{T}} \mathcal{U}^L_t$, $\mathcal{U}^F = \prod_{t \in \mathcal{T}} \mathcal{U}^F_t$
- Exogenous Nature is a product to model multi-stage random variables $\Omega^e = \prod_{t \in \mathcal{T}} \Omega^e_t$
- Expected value replaced by a risk measure over the worst-case scenarios

# W-games can easily deal with multi-leader-multi-follower problems


OPTIMIZATION
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**A multi-leader-follower game for energy demand-side management**

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**ABSTRACT**
A multi-leader-follower game (MLFG) corresponds to a bilevel problem in which the upper level and the lower level are defined by Nash non-cooperative competition among the players acting at the upper level (the leaders) and, at the same time, among the ones acting at the lower level (the followers). MLFGs are known to be complex problems, but they also provide perfect models to describe hierarchical interactions among various actors of real-life problems. In this work, we focus on a class of MLFGs modelling the implementation of demand-side management in an electricity market through price incentives, leading to the so-called *Bilevel Demand-Side Management problem (BDSM)*. Our aim is to propose some innovative reformulations/numerical approaches to efficiently tackle this difficult problem. Our methodology is based on the selection of specific Nash equilibria of the lower level through a precise analysis of the intrinsic characteristics of (BDSM).

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Bilevel optimization; demand-side management; energy markets

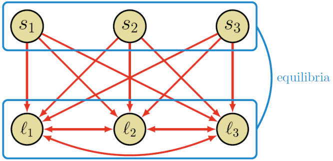


**Figure 1.** Problem scheme for three suppliers (above) and three local agents (below). Arrows represent (possible) energy flows, rectangles stand for Nash games.

Figure: Extracts from [Aussel, Lepaul, and von Niederhäusern, 2022]

- Several leaders and several followers $\mathcal{U}^L = \prod_{l \in L} \mathcal{U}^l$, $\mathcal{U}^F = \prod_{f \in F} \mathcal{U}^f$
- Each player has private knowledge $\Omega^L = \prod_{l \in L} \Omega^l$, $\Omega^F = \prod_{f \in F} \Omega^f$
- Extension of the Nash-Stackelberg equilibrium

# Thank you for listening ;)

- A rich language
- A lot of open questions, and a lot of things not yet properly defined
- We aim to build a unified framework
  to ease the understanding of literature on energy management
- We want to propose a method
  to establish an energy management model from scratch
- We are looking for feedback

Ekaterina Alekseeva, Luce Brotcorne, Sébastien Lepaul, and Alain Montmeat. A bilevel approach to optimize electricity prices. *Yugoslav Journal of Operations Research*, 29:9,30, 2019. doi: 10.2298/yjor171115002a.

D. Aussel, Sébastien Lepaul, and Léonard von Niederhäusern. A multi-leader-follower game for energy demand-side management. *Optimization*, 72:1–31, 05 2022. doi: 10.1080/02331934.2021.1954179.

Patriza Beraldi and s Khodaparasti. A bi-level model for the design of dynamic electricity tariffs with demand-side flexibility. *Soft Computing*, 27, 04 2022. doi: 10.1007/s00500-022-07038-3.

Miguel Carrión, José M Arroyo, and Antonio J Conejo. A bilevel stochastic programming approach for retailer futures market trading. *IEEE Transactions on Power Systems*, 24(3):1446–1456, 2009.